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XXII. *Researches in physical astronomy. By* John William Lubbock, *Esq. Fellow of the Royal Society.*

Read April 29, 1830.

IN the first volume of the Mécanique Céleste, Laplace has given expressions for the variations of the elliptic constants, which are true when the square and higher powers of the disturbing force are neglected; and he has proved, upon the supposition that the planets move in the same direction, in orbits nearly circular and little inclined one to another, that the eccentricities and inclinations vary within small limits, thereby demonstrating within these conditions the stability of the planetary system. But these conditions are not necessary to the stability of a system of bodies, subject to the law of attraction, which obtains in our system. I have given in the following investigation the expressions for the variations of the elliptic constants, which are rigorously true whatever power of the disturbing force be retained; and it is easy to conclude from the form of their expressions, that however far the approximation be carried, the eccentricity, the major axis, and the tangent of the inclination of the orbit to a fixed plane, contain no term which varies with the time; their variations are all periodic, and they oscillate therefore within certain limits. This theorem is no longer true if the planet moves in a resisting medium.

I have also given some equations which obtain when an angle is taken for the independent variable, which in the elliptic movement is the eccentric anomaly, which are of remarkable simplicity, and which, as far as I know, have never been noticed, and the development of the disturbing function R to the quantities involving the squares and products of the eccentricities inclusive.

Let x, y, z denote the rectangular co-ordinates
r . . . . . . distance from the sun
$r^{\backprime}$ . . . . . . distance from the sun projected upon the plane x, y
*λ . . . . . . longitude reckoned upon the plane of its orbit
$\lambda^{\backprime}$ . . . . . . longitude reckoned upon the plane $x\,y$
s . . . . . . tangent of the latitude
v . . . . a variable, which in the elliptic theory is the eccentric anomaly
m . . . . the mass
a . . . . . . semiaxis major
e . . . . . . eccentricity
ϖ . . . . . . longitude of the perihelion
ε . . . . . . longitude of the epoch
ν . . . . . . longitude of the ascending node
ι . . . . . . inclination of the orbit to the plane $x\,y$
α . . . . a constant quantity which accompanies v
of the planet m.

M . . . . the mass of the sun.

$$M + m = \mu, \quad \sqrt{\frac{\mu}{a^3}} = n.$$

$x = r^{\backprime}\cos\lambda^{\backprime}$, $y = r^{\backprime}\sin\lambda^{\backprime}$, $z = r^{\backprime}s$, and in the elliptic motion $r^{\backprime 2}\mathrm{d}\lambda^{\backprime} = h\,\mathrm{d}\,t$.

$$R = m_{\prime}\left\{\frac{x\,x_{\prime} + y\,y_{\prime} + z\,z_{\prime}}{\{x_{\prime}^2 + y_{\prime}^2 + z_{\prime}^2\}^{\frac{3}{2}}} - \frac{1}{\{(x - x_{\prime})^2 + (y - y_{\prime})^2 + (z - z_{\prime})^2\}^{\frac{1}{2}}}\right\}$$

$$= m_{\prime}\left\{\frac{r^{\backprime}\{\cos(\lambda^{\backprime} - \lambda^{\backprime}_{\prime}) + s\,s_{\prime}\}}{r^{\backprime 2}_{\prime}(1 + s_{\prime}^2)^{\frac{3}{2}}} - \frac{1}{\left\{r^{\backprime 2}(1+s^2) - 2r^{\backprime}r^{\backprime}_{\prime}\{\cos(\lambda^{\backprime} - \lambda^{\backprime}_{\prime}) + s\,s_{\prime}\} + r^{\backprime 2}_{\prime}(1 + s_{\prime}^2)\right\}^{\frac{1}{2}}}\right\}$$

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* LAPLACE uses the letter v to denote longitude, u the eccentric anomaly, and φ the inclination of the orbit to a fixed plane; but as v is very frequently used to signify velocity, and φ geographical latitude, and as the letters of the Greek alphabet are generally used for angles, I have taken the letters λ, v, and ι for these quantities.

$$= -m_{\prime}\left\{\begin{aligned}&\frac{1}{r_{\prime}^{\backprime}\sqrt{1+s_{\prime}^{2}}}+\frac{r^{\backprime 2}}{4r_{\prime}^{\backprime 3}}\{1+3\cos(2\lambda^{\backprime}-2\lambda_{\prime}^{\backprime})+12\,s\,s_{\prime}\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})-2s^{2}\}\\&\qquad+\frac{r^{\backprime 3}}{8r_{\prime}^{\backprime 4}}\{3(1-4s^{2})\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})+5\cos(3\lambda^{\backprime}-3\lambda_{\prime}^{\backprime})\}\\&\qquad+\frac{r^{\backprime 4}}{64r_{\prime}^{\backprime 5}}\{9+20\cos(2\lambda^{\backprime}-2\lambda_{\prime}^{\backprime})+35\cos(4\lambda^{\backprime}-4\lambda_{\prime}^{\backprime})\}\end{aligned}\right\}$$

$$\left(\frac{dR}{dr^{\backprime}}\right)=m_{\prime}\left\{\frac{\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})+s\,s_{\prime}}{r_{\prime}^{\backprime 2}(1+s_{\prime}^{2})^{\frac{3}{2}}}+\frac{r^{\backprime}(1+s^{2})-r_{\prime}^{\backprime}\{\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})+s\,s_{\prime}\}}{\{r^{\backprime 2}(1+s^{2})-2r^{\backprime}r_{\prime}^{\backprime}\{\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})+s\,s_{\prime}\}+r_{\prime}^{\backprime 2}(1+s_{\prime}^{2})\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{dR}{d\lambda}\right)=-m_{\prime}\left\{\frac{r^{\backprime}\sin\lambda^{\backprime}-\lambda_{\prime}^{\backprime}}{r_{\prime}^{\backprime 2}(1+s^{\prime 2})^{\frac{3}{2}}}-\frac{r^{\backprime}r_{\prime}^{\backprime}\sin(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})}{\{r^{\backprime 2}(1+s^{2})-2r^{\backprime}r_{\prime}^{\backprime}\{\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})+s\,s_{\prime}\}+r_{\prime}^{\backprime 2}(1+s_{\prime}^{2})\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{dR}{ds}\right)=m_{\prime}\left\{\frac{r^{\backprime}s_{\prime}}{r_{\prime}^{\backprime 2}(1+s_{\prime}^{2})^{\frac{3}{2}}}+\frac{r^{\backprime 2}s-r^{\backprime}r_{\prime}^{\backprime}s_{\prime}}{\{r^{\backprime 2}(1+s^{2})-2r^{\backprime}r_{\prime}^{\backprime}\{\cos(\lambda^{\backprime}-\lambda_{\prime}^{\backprime})+s\,s_{\prime}\}+r_{\prime}^{\backprime 2}(1+s_{\prime}^{2})\}^{\frac{3}{2}}}\right\}$$

Let P be the place of the planet m, S the place of the sun, S N the intersection of the orbits of m and $m_{\prime}$, S L the line from which longitudes are reckoned, $\mathrm{P}_{\prime}^{\backprime}$, the projection of $\mathrm{P}_{\prime}$ upon the plane of the orbit of P; then if the plane $x\,y$ coincide with the orbit of m, $\mathrm{SP}=r$, $\mathrm{PSL}=\lambda$, $\mathrm{NSL}=\nu$, $\mathrm{P}_{\prime}\mathrm{SN}+\mathrm{NSL}=\lambda_{\prime}$, $\mathrm{P}_{\prime}^{\backprime}\mathrm{SL}=\lambda_{\prime}^{\backprime}$, $\mathrm{SP}_{\prime}^{\backprime}=r_{\prime}^{\backprime}$, $\mathrm{SP}_{\prime}=r_{\prime}=r_{\prime}^{\backprime}(1+s_{\prime}^{2})^{\frac{1}{2}}$

$$R=m_{\prime}\left\{\frac{\mathrm{SP}\times\mathrm{SP}_{\prime}\cos\mathrm{PSP}_{\prime}}{\mathrm{SP}_{\prime}^{3}}-\frac{1}{\{\mathrm{SP}^{2}-2\,\mathrm{SP}\times\mathrm{SP}_{\prime}\cos\mathrm{PSP}_{\prime}+\mathrm{SP}_{\prime}^{2}\}^{\frac{1}{2}}}\right\}$$

$$\left(\frac{dR}{dr}\right)=m_{\prime}\left\{\frac{\mathrm{SP}_{\prime}^{\backprime}\cos\mathrm{PSP}_{\prime}^{\backprime}}{\mathrm{SP}_{\prime}^{3}}+\frac{\mathrm{SP}-\mathrm{SP}_{\prime}^{\backprime}\cos\mathrm{PSP}_{\prime}^{\backprime}}{\{\mathrm{SP}^{2}-2\,\mathrm{SP}\times\mathrm{SP}_{\prime}^{\backprime}\cos\mathrm{PSP}_{\prime}^{\backprime}+\mathrm{SP}_{\prime}^{2}\}^{\frac{3}{2}}}\right\}$$

$$r\left(\frac{dR}{dr}\right)=m_{\prime}\left\{\frac{\mathrm{SP}\times\mathrm{SP}_{\prime}\cos\mathrm{PSP}_{\prime}}{\mathrm{SP}_{\prime}^{3}}+\frac{\mathrm{SP}^{2}-\mathrm{SP}\times\mathrm{SP}_{\prime}\cos\mathrm{PSP}_{\prime}}{\{\mathrm{SP}^{2}-2\,\mathrm{SP}\times\mathrm{SP}_{\prime}^{\backprime}\cos\mathrm{PSP}_{\prime}^{\backprime}+\mathrm{SP}_{\prime}^{2}\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{dR}{d\lambda}\right)=-m_{\prime}\left\{\frac{\mathrm{SP}\times\mathrm{SP}_{\prime}^{\backprime}\sin\mathrm{PSP}_{\prime}^{\backprime}}{\mathrm{SP}_{\prime}^{3}}-\frac{\mathrm{SP}\times\mathrm{SP}_{\prime}^{\backprime}\sin\mathrm{PSP}_{\prime}^{\backprime}}{\{\mathrm{SP}^{2}-2\,\mathrm{SP}\times\mathrm{SP}_{\prime}^{\backprime}\cos\mathrm{PSP}_{\prime}^{\backprime}+\mathrm{SP}_{\prime}^{2}\}^{\frac{3}{2}}}\right\}$$

$$\left(\frac{dR}{ds}\right)=m_{\prime}\left\{\frac{\mathrm{SP}\times\mathrm{P}_{\prime}\mathrm{P}_{\prime}^{\backprime}}{\mathrm{SP}_{\prime}^{3}}-\frac{\mathrm{SP}\times\mathrm{P}_{\prime}\mathrm{P}_{\prime}^{\backprime}}{\{\mathrm{SP}^{2}-2\,\mathrm{SP}\times\mathrm{SP}_{\prime}^{\backprime}\cos\mathrm{PSP}_{\prime}^{\backprime}+\mathrm{SP}_{\prime}^{2}\}^{\frac{3}{2}}}\right\}$$

$$\begin{aligned}\cos(\lambda-\lambda_{\prime})&=\cos(\lambda-\nu)\cos(\lambda_{\prime}-\nu)+\cos\iota\sin(\lambda-\nu)\sin(\lambda_{\prime}-\nu)\\&=\cos^{2}\frac{\iota}{2}\cos(\lambda-\lambda_{\prime})+\sin^{2}\frac{\iota}{2}\cos(\lambda+\lambda_{\prime}-2\nu)\end{aligned}$$

$$\tan(\lambda_{\prime}^{\backprime}-\nu)=\cos\iota\tan(\lambda_{\prime}-\nu)$$

$$\sin(\lambda_{\prime}^{\backprime}-\nu)=\frac{\cos\iota\tan(\lambda_{\prime}-\nu)}{(1+\cos^{2}\iota\tan^{2}(\lambda_{\prime}-\nu))^{\frac{1}{2}}}$$

$$\cos(\lambda'_{,}-\nu)=\frac{1}{(1+\cos^2\iota\tan^2(\lambda_{,}-\nu))^{\frac{1}{2}}}$$

$$r\,r'_{,}\sin(\lambda-\lambda'_{,})=r\,r'_{,}\{\sin\lambda\cos\lambda'_{,}-\cos\lambda\sin\lambda_{,}\}$$

$$\cos\lambda'_{,}=\cos(\lambda'_{,}-\nu)\cos\nu-\sin(\lambda'_{,}-\nu)\sin\nu$$

$$=\frac{\cos(\lambda_{,}-\nu)\cos\nu-\sin(\lambda_{,}-\nu)\sin\nu\left(1-2\sin^2\frac{\iota}{2}\right)}{(1-\sin^2\iota\sin^2(\lambda_{,}-\nu)^{\frac{1}{2}}}$$

$$=\frac{\cos\lambda_{,}+2\sin^2\frac{\iota}{2}\sin(\lambda_{,}-\nu)\sin\nu}{(1-\sin^2\iota\sin^2(\lambda_{,}-\nu))^{\frac{1}{2}}}$$

$$\sin\lambda'_{,}=\sin(\lambda'_{,}-\nu)\cos\nu+\cos(\lambda'_{,}-\nu)\sin\nu$$

$$=\frac{\sin(\lambda_{,}-\nu)\cos\nu\left(1-2\sin^2\frac{\iota}{2}\right)+\cos(\lambda_{,}-\nu)\sin\nu}{(1-\sin^2\iota\sin^2(\lambda_{,}-\nu))^{\frac{1}{2}}}$$

$$\sin \mathrm{P}_{,}\mathrm{S}\,\mathrm{P}'_{,}=\sin\iota\sin(\lambda_{,}-\nu),\quad r'_{,}=r_{,}\cos \mathrm{P}_{,}\mathrm{S}\,\mathrm{P}'_{,}=r_{,}\left(1-\sin^2\iota\sin^2(\lambda_{,}-\nu)\right)^{\frac{1}{2}}$$

therefore,

$$r\,r'_{,}\sin(\lambda-\lambda'_{,})=r\,r_{,}\left\{\sin(\lambda-\lambda_{,})+2\sin^2\frac{\iota}{2}\sin(\lambda_{,}-\nu)\cos(\lambda-\nu)\right\}$$

$$=r\,r_{,}\left\{\cos^2\frac{\iota}{2}\sin(\lambda-\lambda_{,})+\sin^2\frac{\iota}{2}\sin(\lambda+\lambda_{,}-2\nu)\right\}$$

similarly it may be shown that

$$r\,r'_{,}\cos(\lambda-\lambda'_{,})=r\,r_{,}\left\{\cos^2\frac{\iota}{2}\cos(\lambda-\lambda_{,})+\sin^2\frac{\iota}{2}\cos(\lambda+\lambda_{,}-2\nu)\right\}$$

$$\mathrm{S}\,\mathrm{P}_{,}=\frac{a_{,}(1-e_{,}^2)}{1+e_{,}\cos(\mathrm{P}_{,}\mathrm{S}\,\mathrm{N}+\mathrm{N}\,\mathrm{S}\,\mathrm{L}-\varpi_{,})}=\frac{a_{,}(1-e_{,}^2)}{1+e_{,}\cos(\lambda_{,}-\varpi_{,})}$$

$$\frac{d^2x}{dt^2}+\frac{\mu x}{r^3}+\left(\frac{dR}{dx}\right)=0,\quad \frac{d^2y}{dt^2}+\frac{\mu y}{r^3}+\left(\frac{dR}{dy}\right)=0,\quad \frac{d^2z}{dt^2}+\frac{\mu z}{r^3}+\left(\frac{dR}{dz}\right)=0$$

$$\frac{r'^2d\lambda'^2+dr'^2(1+s^2)+2r's\,dr'ds+r'^2ds^2}{dt^2}-\frac{2\mu}{r'(1+s^2)^{\frac{1}{2}}}+\frac{\mu}{a}+2\int dR=0$$

d R being the differential of R with regard only to the co-ordinates of the planet m.

$$\frac{d^2r'(1+s^2)-r'd\lambda'^2+2s\,dr'ds+r's\,d^2s}{dt^2}+\frac{\mu}{r'(1+s^2)^{\frac{1}{2}}}+\left(\frac{dR}{dr'}\right)=0$$

$$\frac{d.\frac{r'^2d\lambda'}{dt}}{dt}+\left(\frac{dR}{d\lambda'}\right)=0$$

$$\frac{r's\,d^2r'+2r'dr'ds+r'^2d^2s}{dt^2}+\frac{s}{r'(1+s^2)^{\frac{3}{2}}}+\left(\frac{dR}{ds}\right)=0$$

$$\frac{r^{\backprime}\,\mathrm{d}^2 r^{\backprime} - r^{\backprime 2}\,\mathrm{d}\,\lambda^{\backprime 2}}{\mathrm{d}\,t^2} + \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{1}{2}}} + r^{\backprime}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) - s\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) = 0$$

$$\frac{r^{\backprime 2}\,\mathrm{d}^2 s + 2\,r^{\backprime}\,\mathrm{d}\,r^{\backprime}\,\mathrm{d}\,s - r^{\backprime 2} s\,\mathrm{d}\,\lambda^{\backprime 2}}{\mathrm{d}\,t^2} + (1+s^2)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) - r^{\backprime}\,s\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) = 0$$

$$\frac{\mathrm{d}^2 . r^{\backprime 2}(1+s^2)}{2\,\mathrm{d}\,t^2} - \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{1}{2}}} + \frac{\mu}{a} + 2\int \mathrm{d}\,R + r^{\backprime}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) = 0$$

Making λ the independent variable instead of *t*,

$$\frac{\mathrm{d}^2 r^{\backprime}}{\mathrm{d}\,t^2} - 2\,\frac{\mathrm{d}\,r^{\backprime}\,\mathrm{d}^2 t}{\mathrm{d}\,t^3} - r^{\backprime 2}\,\frac{\mathrm{d}\,\lambda^{\backprime 2}}{\mathrm{d}\,t^2} + \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{1}{2}}} + r^{\backprime}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) - s\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) = 0$$

$$r^{\backprime 4}\,\frac{\mathrm{d}\,\lambda^{\backprime 2}}{\mathrm{d}\,t^2} = h^2 - 2\int r^{\backprime 2}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,\lambda^{\backprime},$$ *h* being a constant,

$$4\,r^{\backprime 3}\,d\,r^{\backprime}\,\frac{\mathrm{d}\,\lambda^{\backprime 2}}{\mathrm{d}\,t^2} - 2\,r^{\backprime 4}\,\frac{\mathrm{d}\,\lambda^{\backprime 2}\,\mathrm{d}^2 t}{\mathrm{d}\,t^3} = -\,2\,r^{\backprime 2}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,\lambda^{\backprime}$$

$$\left\{\frac{\mathrm{d}^2 . \frac{1}{r^{\backprime}}}{\mathrm{d}\,\lambda^{\backprime 2}} + \frac{1}{r^{\backprime}}\right\}\left\{1 - \frac{2}{h^2}\int r^{\backprime 2}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,\lambda^{\backprime}\right\} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}$$

$$-\,\frac{r^{\backprime}}{h^2}\left\{r^{\backprime}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) - s\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) - \frac{1}{r^{\backprime}}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\frac{\mathrm{d}\,r^{\backprime}}{\mathrm{d}\,\lambda^{\backprime}}\right\} = 0$$

$$\left\{\frac{\mathrm{d}^2 s}{\mathrm{d}\lambda^{\backprime 2}} + s\right\}\left\{1 - \frac{2}{h^2}\int r^{\backprime 2}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\lambda^{\backprime}\right\}$$

$$+\,\frac{r^{\backprime 2}}{h^2}\left\{(1+s^2)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) - r^{\backprime}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) - \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\frac{\mathrm{d}\,s}{\mathrm{d}\lambda^{\backprime}}\right\} = 0$$

When the disturbing force is neglected

$$\frac{\mathrm{d}.r^{\backprime 2}\mathrm{d}\lambda^{\backprime}}{\mathrm{d}\,t^{\backprime}} = 0, \quad \frac{\mathrm{d}^2 . \frac{1}{r^{\backprime}}}{\mathrm{d}\,\lambda^{\backprime 2}} + \frac{1}{r^{\backprime}} - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} = 0, \quad \frac{\mathrm{d}^2 s}{\mathrm{d}\lambda^{\backprime 2}} + s = 0$$

of which equations the integrals are,

$$r^{\backprime 2}\,\mathrm{d}\,\lambda^{\backprime} = h\,\mathrm{d}\,t, \quad \frac{1}{r^{\backprime}} = \frac{\mu\cos\iota^2}{h^2}\left\{(1+s^2)^{\frac{1}{2}} + e\cos(\lambda^{\backprime} - \varpi)\right\}$$

$$s = \tan\iota\,\sin(\lambda^{\backprime} - \nu).$$

If $\mathrm{d}\,t = \sqrt{\frac{a}{\mu}}\,r^{\backprime}\,\mathrm{d}\,v$, and v be taken for the independent variable

$$r^{\backprime}\,\frac{\mathrm{d}^2 r^{\backprime}}{\mathrm{d}\,t^2} - r^{\backprime}\,\mathrm{d}\,r^{\backprime}\,\frac{\mathrm{d}^2 t}{\mathrm{d}\,t^3} - r^{\backprime 2}\,\frac{\mathrm{d}\,\lambda^{\backprime 2}}{\mathrm{d}\,t^2} + \frac{\mu}{r^{\backprime}(1+s^2)^{\frac{1}{2}}} + r^{\backprime}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right) - s\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right) = 0$$

$$\mathrm{d}^2 t = \left(\frac{a}{\mu}\right)^{\frac{1}{2}} \mathrm{d}\, r^{\backprime}\, \mathrm{d}\, v$$

$$\frac{\mathrm{d}^2 r^{\backprime}}{\mathrm{d}\, v^2} + \left(\frac{\mathrm{d}.r^{\backprime} s}{\mathrm{d}\, v}\right)^2 - \frac{a}{(1+s^2)^{\frac{1}{2}}} + r^{\backprime} + \frac{a r^{\backprime}}{\mu}\left\{2\int \mathrm{d}\, R + r^{\backprime}\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r^{\backprime}}\right) - s\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, s}\right)\right\} = 0.$$

If the orbit of the planet m coincide with the plane $x\, y$, s is of the order of the disturbing force, of which therefore neglecting the square, $r^{\backprime} = r$, $\lambda^{\backprime} = \lambda$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, v^2} - a + r + \frac{a\, r}{\mu}\left\{ 2\int \mathrm{d}\, R + r\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right)\right\} = 0.$$

When the disturbing force is neglected, the integral of this equation is $r = a\,\{1 - e \cos\,(v - \alpha)\}$, v being the eccentric anomaly.

$$n\, t + \varepsilon - \varpi = v - \alpha - e \sin\,(v - \alpha)$$

$$\tan \frac{\lambda - \varpi}{2} = \left\{\frac{1+e}{1-e}\right\}^{\frac{1}{2}} \tan \frac{v - \alpha}{2}.$$

If Q be put for the quantity $\frac{a\, r}{\mu}\left\{ 2\int \mathrm{d}\, R + r\left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right)\right\}$ and the constant α, which may afterwards be replaced, be omitted for the present,

$$r = a\,\{1 - e \cos v\} - \sin v \int Q \cos v\, \mathrm{d}\, v + \cos v \int Q \sin v\, \mathrm{d}\, v$$

$$\mathrm{d}\, t = \sqrt{\frac{a}{\mu}}\left\{a\,\{1 - e\cos v\} - \sin v \int Q \cos v\, \mathrm{d}\, v + \cos v \int Q \sin v\, \mathrm{d}\, v\right\} \mathrm{d}\, v$$

$$n\, t + \varepsilon - \varpi = v - e \sin v - \frac{1}{a}\left\{\int Q\, \mathrm{d}\, v - \cos v \int Q \cos v\, \mathrm{d}\, v - \sin v \int Q \sin v\, \mathrm{d}\, v\right\}$$

If $v = \mathrm{f}\,(n\, t + \varepsilon - \varpi)$ in the elliptic theory, then neglecting the square of the disturbing force,

$$v = \mathrm{f}\,(n t + \varepsilon - \varpi) + \frac{\mathrm{d.f}(n t + \varepsilon - \varpi)}{\mathrm{d}\, t}\,\frac{1}{a}\left\{\int Q\, \mathrm{d} v - \cos v \int Q \cos v\, \mathrm{d} v - \sin v \int Q \sin v\right\} \mathrm{d} v$$

If $\delta\, v$, $\delta\, r$ denote the values of those parts of r and v which are due to the first power of the disturbing force,

$$\delta\, v = \frac{\mathrm{d}\, v}{a\, n\, \mathrm{d}\, t}\left\{\int Q\, \mathrm{d}\, v - \cos v \int Q \cos v\, \mathrm{d}\, v - \sin v \int Q \sin v\, \mathrm{d}\, v\right\}$$

$$\delta\, r = a\, e \sin v\, \delta\, v - \sin v \int Q \cos v\, \mathrm{d}\, v + \cos v \int Q \sin v\, \mathrm{d}\, v$$

$$= \frac{\sin v}{1 - e \cos v} \int Q\,\{e - \cos v\}\, \mathrm{d}\, v - \frac{e - \cos v}{1 - e\cos v} \int Q \sin v\, \mathrm{d}\, v.$$

In the elliptic theory,

$$\mathrm{d}\, v = \frac{n\, \mathrm{d}\, t}{1 - e \cos v}, \qquad \frac{\cos v - e}{1 - e \cos v} = \cos \lambda, \qquad \frac{\sin v\,(1 - e^2)^{\frac{1}{2}}}{1 - e \cos v} = \sin \lambda$$

therefore,

$$\delta r = \frac{a\,n \cos\lambda \int r \sin\lambda \left\{ 2\int \mathrm{d} R + r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) \right\} \mathrm{d} t - a\,n \sin\lambda \int r \cos\lambda \left\{ 2 \int \mathrm{d} R + r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) \right\} \mathrm{d} t \Big\}}{\mu\,(1 - e^2)^{\frac{1}{2}}}$$

which is the equation X of the Mécanique Céleste, vol. i. p. 258.

Multiplying the equation of p. 114, l. 2, by $\frac{2r'^2 \mathrm{d}\lambda'}{\mathrm{d} t}$, and integrating, $\frac{r'^4 \mathrm{d}\lambda'^2}{\mathrm{d} t^2} + 2\int r'^2 \left(\frac{\mathrm{d R}}{\mathrm{d}\lambda'}\right) \mathrm{d}\lambda' = h_0^2$, if $h^2 = h_0^2 - 2\int r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d}\lambda'}\right) \mathrm{d}\lambda'$, h being variable, and h_0 the value of h at a given epoch, $h\,\mathrm{d} h = - r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d}\lambda'}\right) \mathrm{d}\lambda'$, $r^2\,\mathrm{d}\lambda' = h\,\mathrm{d}\,t$, and making λ' the independent variable instead of t, $2\,r'\,\mathrm{d}\,r'\,\mathrm{d}\,\lambda' = \mathrm{d}\,h\,\mathrm{d}\,t + h\,\mathrm{d}^2\,t$

$$r' \frac{\mathrm{d}^2 r'}{\mathrm{d} t^2} - \frac{r'\mathrm{d} r'\mathrm{d}^2 t}{\mathrm{d} t^3} - \frac{r^2 \mathrm{d}\lambda'^2}{\mathrm{d} t^2} + \frac{\mu}{r'(1+s^2)^{\frac{3}{2}}} + r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) = 0$$

$$r' \left(\frac{\mathrm{d}^2 r'}{\mathrm{d} t^2}\right) + \frac{r'\mathrm{d} r'\mathrm{d} t}{h\,\mathrm{d} t^2} - \frac{2\,r'^2 \mathrm{d} r'^2 \mathrm{d}\lambda}{h\,\mathrm{d} t^3} - \frac{r'^2 \mathrm{d}\lambda'^2}{\mathrm{d} t^2} + \frac{\mu}{r'(1 + 3^2)^{\frac{3}{2}}} + r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) = 0$$

$$\frac{\mathrm{d}^2 . \frac{1}{r'}}{\mathrm{d}\lambda'^2} + \frac{1}{r'} - \frac{\mu}{h^2 (1+s^2)^{\frac{3}{2}}} - \frac{r'}{h^2} \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d}\lambda'}\right) \frac{\mathrm{d} r'}{\mathrm{d}\lambda'} \right\} = 0$$

$$\frac{\mathrm{d}^2 s}{\mathrm{d}\lambda'^2} + s + \frac{r'^2}{h^2} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d}\lambda'}\right) \frac{\mathrm{d} s'}{\mathrm{d}\lambda'} \right\} = 0$$

If all the constants in the elliptic integrals are supposed to vary, subject to the condition that they still satisfy these differential equations, and that the form of the first differential coefficients $\frac{\mathrm{d} r'}{\mathrm{d}\lambda'}$, $\frac{\mathrm{d} s}{\mathrm{d}\lambda'}$, remains unaltered,

$$\frac{\mathrm{d} . \frac{1}{r'}}{\mathrm{d}\lambda'} = \frac{\mu \cos\iota^2}{h^2} \left\{ \frac{s}{(1+s^2)^{\frac{1}{2}}} \frac{\mathrm{d} s}{\mathrm{d}\lambda'} - e \sin(\lambda' - \varpi) \right\}, \qquad \frac{\mathrm{d} s}{\mathrm{d}\lambda'} = \tan\iota \cos(\lambda' - \nu)$$

$$- 2 \{(1 + s^2)^{\frac{1}{2}} + e \cos(\lambda' - \varpi)\} \{h^2 \sin\iota\, \mathrm{d}\,\iota + \cos\iota\, h\, \mathrm{d}\, h\}$$

$$+ h^2 \cos\iota \cos(\lambda' - \varpi)\, \mathrm{d}\, e + h^2 \cos\iota\, e \sin(\lambda' - \varpi)\, \mathrm{d}\,\varpi = 0$$

$$- 2 \left\{ \frac{s \tan\iota \cos(\lambda' - \nu)}{(1+s^2)^{\frac{1}{2}}} - e \sin(\lambda' - \varpi) \right\} \{h^2 \sin\iota\, \mathrm{d}\,\iota + \cos\iota\, h\, \mathrm{d}\, h\}$$

$$- h^2 \cos\iota \sin(\lambda' - \varpi)\, \mathrm{d}\, e + h^2 \cos\iota\, e \cos(\lambda' - \varpi)\, \mathrm{d}\,\varpi$$

$$- \frac{\cos\iota\, s\, r'^2}{(1+s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d}\lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d}\lambda'}\right) \right\} \mathrm{d}\,\lambda'$$

$$- \frac{h^2 r'}{\mu \cos \iota} \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$\sin (\lambda' - \nu) \frac{\mathrm{d} \iota}{\cos \iota^2} - \tan \iota \cos (\lambda' - \nu) \mathrm{d} \nu = 0$$

$$\cos (\lambda' - \nu) \frac{\mathrm{d} \iota}{\cos \iota^2} + \tan \iota \sin (\lambda' - \nu) \mathrm{d} \nu + \frac{r'^2}{h^2} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$h \, \mathrm{d} h = - r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \mathrm{d} \lambda'$$

Whence by elimination,

$$h^2 \cos \iota \, \mathrm{d} e + 2 \left\{ (1 + s^2)^{\frac{1}{2}} \cos (\lambda' - \varpi) + e - \frac{\tan \iota \, s \cos (\lambda' - \nu) \sin (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \right\}$$

$$\left\{ \sin \iota \cos^2 \iota \, r'^2 \cos (\lambda' - \nu) \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} + \cos \iota \, r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$+ \frac{s \cos \iota \, r'^2 \sin (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$+ \frac{h^2 r'}{\mu \cos \iota} \sin (\lambda' - \varpi) \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$h^2 \cos \iota \, e \, \mathrm{d} \varpi + 2 \left\{ (1 + s^2)^{\frac{1}{2}} \sin (\lambda' - \varpi) + \frac{s \tan \iota \cos (\lambda' - \nu) \cos (\lambda' - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \right\}$$

$$\left\{ \sin \iota \cos^2 \iota \, r'^2 \cos (\lambda' - \nu) \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} + \cos \iota \, r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$- \frac{s \cos \iota \, r'^2 \cos (\lambda - \varpi)}{(1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda'$$

$$- \frac{h^2 r'}{\mu \cos \iota} \cos (\lambda' - \varpi) \left\{ r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - \frac{1}{r'} \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$d \iota + \frac{r'^2 \cos \iota^2 \cos (\lambda' - \nu)}{h^2} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} r'}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

$$d \nu + \frac{r'^2 \sin (\lambda' - \nu)}{h^2 \tan \iota} \left\{ (1 + s^2) \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) - s \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) - \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \left(\frac{\mathrm{d} s}{\mathrm{d} \lambda'}\right) \right\} \mathrm{d} \lambda' = 0$$

If $x = \frac{r \cos \lambda'}{(1 + s^2)^{\frac{1}{2}}}$, $y = \frac{r \sin \lambda'}{(1 + s^2)^{\frac{1}{2}}}$, $z = \frac{r s}{(1 + s^2)^{\frac{1}{2}}}$,

$$\frac{\mathrm{d}\, r^2 + \frac{r^2}{1 + s^2}\left(\frac{\mathrm{d}\, s^2}{1 + s^2} + \mathrm{d}\, \lambda'^2\right)}{\mathrm{d}\, t^2} - \frac{2 \mu}{r} + \frac{\mu}{a} + 2 \int \mathrm{d}\, R = 0$$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, t^2} - \frac{r}{1 + s^2} \frac{\left(\frac{\mathrm{d}\, s^2}{1 + s^2} + \mathrm{d}\, \lambda'^2\right)}{\mathrm{d}\, t^2} + \frac{\mu}{r^2} + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right) = 0$$

$$\frac{\mathrm{d} \,.\, \frac{r^2}{1 + s^2} \,.\, \frac{\mathrm{d}\, \lambda'}{\mathrm{d}\, t}}{\mathrm{d}\, t} + \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, \lambda'}\right) = 0$$

$$\frac{r^2\, \mathrm{d}^2 s + 2\, r\, \mathrm{d}\, r\, \mathrm{d}\, s - \frac{2\, r^2 s\, \mathrm{d}\, s^2}{(1 + s^2)} + r^2 s\, \mathrm{d}\, \lambda'^2}{\mathrm{d}\, t^2} + (1 + s^2)^2 \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, s}\right) = 0$$

Of which equations the integrals are

$$\frac{r^2}{1 + s^2} \,.\, \mathrm{d}\, \lambda' = h\, \mathrm{d}\, t, \quad \frac{1}{r} = \frac{\mu \cos \iota^2}{h^2 (1 + s^2)^{\frac{1}{2}}} \left\{ (1 + s^2)^{\frac{1}{2}} + e \cos (\lambda' - \varpi) \right\}$$

$$s = \tan \iota \sin (\lambda' - \nu)$$

If $\mathrm{d}\, t = \sqrt{\frac{a}{\mu}}\, r\, \mathrm{d}\, v$, and v be taken for the independent variable

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, v^2} - a + r + \frac{a r}{\mu} \left\{ 2 \int \mathrm{d}\, R + r \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right) \right\} = 0$$

$r = a \{1 - e' \cos (v - \alpha)\}$ in the elliptic motion.

e' is accented for the present in order to distinguish it from e.

If the constants in the elliptic integrals are supposed to vary, subject to the condition that they still satisfy these differential equations, and that the form of the first differential coefficient $\frac{\mathrm{d}\, r}{\mathrm{d}\, v}$ remains unaltered,

$$\mathrm{d}^2 t = \frac{r\, \mathrm{d}\, v\, \mathrm{d}\, a}{2 \sqrt{a \mu}} + \sqrt{\frac{a}{\mu}}\, \mathrm{d}\, r\, \mathrm{d}\, v, \quad \frac{r\, \mathrm{d}^2 r}{\mathrm{d}\, t^2} = \frac{r\, \mathrm{d}\, r^2}{\mathrm{d}\, t^3} - \frac{r\, \mathrm{d}\, r\, \mathrm{d}^2 t}{\mathrm{d}\, t^3}$$

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\, v^2} - a + r + \frac{a r^2}{\mu} \left(\frac{\mathrm{d}\, R}{\mathrm{d}\, r}\right) - \frac{\mathrm{d}\, r\, \mathrm{d}\, a}{2\, a\, \mathrm{d}\, v^2} = 0$$

$$\big(1 - e' \cos (v - \alpha)\big)\, \mathrm{d}\, a - a \cos (v - \alpha)\, \mathrm{d}\, e' - a\, e' \sin (v - \alpha)\, \mathrm{d}\, \alpha = 0$$

$$e' \sin (v - \alpha)\, \mathrm{d}\, a + a \sin (v - \alpha)\, \mathrm{d}\, e' - a\, e' \cos (v - \alpha)\, \mathrm{d}\, \alpha$$

$$+\frac{a\,r^2}{\mu}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,\upsilon+\frac{a^2\,e}{\mu}\sin\,(\upsilon-\alpha)\,\mathrm{d}\,R=0$$

$$\mathrm{d}\,a=-\frac{2\,a^2}{\mu}\,\mathrm{d}\,R$$

$$\mathrm{d}\,e'=\frac{a}{\mu}\{2\,e'-2\cos\,(\upsilon-\alpha)-e'\sin\,(\upsilon-\alpha)^2\}\,\mathrm{d}\,R-\frac{r^2}{\mu}\sin\,(\upsilon-\alpha)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,\upsilon$$

$$e\,\mathrm{d}\,\alpha=\frac{a}{\mu}\sin\,(\upsilon-\alpha)\,\{e'\cos\,(\upsilon-\alpha)-2\}\,\mathrm{d}\,R+\frac{r^2}{\mu}\cos\,(\upsilon-\alpha)\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,\upsilon$$

$$\int n\,\mathrm{d}\,t+\varepsilon-\varpi=\upsilon-\alpha-e'\sin\,(\upsilon-\alpha)$$

$$\mathrm{d}\,\varepsilon-\mathrm{d}\,\varpi=-\,\mathrm{d}\,\alpha-\sin\,(\upsilon-\alpha)\,\mathrm{d}\,e'+e'\cos\,(\upsilon-\alpha)\,\mathrm{d}\,\alpha.$$

The equation of condition which obtains between the constants a, e, ϖ, ν, ι and h may be found from the equation

$$\frac{\mathrm{d}\,r^2+\dfrac{r^2}{1+s^2}\left(\dfrac{\mathrm{d}\,s^2}{1+s^2}+\mathrm{d}\,\lambda'^2\right)}{\mathrm{d}\,t^2}-\frac{2\,\mu}{r}+\frac{\mu}{a}=0$$

$$\frac{\mathrm{d}\,r^2}{\mathrm{d}\,t^2}+\frac{h^2}{r^2\cos\iota^2}-\frac{2\,\mu}{r}+\frac{\mu}{a}=0$$

which gives

$$\frac{\mu\cos\iota^2}{h^2}\left\{e^2\cos^2\iota\sin^2(\nu-\varpi)-\{1+e\cos(\nu-\varpi)\}\{1-e\cos(\nu-\varpi)\}\right\}+\frac{1}{a}=0$$

Equating the values of r which have been found,

$$\frac{h^2\sqrt{1+s^2}}{\mu\cos\iota^2\{\sqrt{1+s^2}+e\cos(\lambda-\varpi)\}}=a\,\{1-e'\cos\,(\upsilon-\alpha)\}$$

since the origin of the angle υ is arbitrary, we may suppose $\lambda-\varpi=0$, and $\upsilon-\alpha=0$ at the same time,

so that $\dfrac{h^2\sqrt{1+\tan\iota^2\sin(\varpi-\nu)^2}}{\mu\cos\iota^2\{\sqrt{1+\tan\iota^2\sin(\varpi-\nu)^2}+e\}}=a\,(1-e')$

All the equations which have hitherto been proved are rigorously true, whatever powers of the disturbing function be retained. They are susceptible of simplification when the square and higher powers of the disturbing function are neglected: in this case, if the orbit be supposed to coincide with the plane $x\,y$, $\tan\iota=0$, and if the longitude be reckoned from the perihelion of the planet P,

$$h^2\,\mathrm{d}\,e+\{2\cos\lambda+e+e\cos\lambda^2\}\,r^2\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)\mathrm{d}\,\lambda+\frac{h^2r^2\sin\lambda}{\mu}\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d}\,\lambda=0$$

$$h^2\, e\, \mathrm{d}\varpi + \{2 + \cos\lambda\} \sin\lambda\, r^2 \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) \mathrm{d}\lambda - h^2 r^2 \cos\lambda \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \mathrm{d}\lambda = 0$$

$$\mathrm{d}\iota + r^2 \frac{\cos(\lambda - \nu)}{\mu} \left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) \mathrm{d}\lambda = 0$$

$$\mathrm{d}\nu + r^2 \frac{\sin(\lambda - \nu)}{\mu} \left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) \mathrm{d}\lambda = 0$$

by equation, p. 336, line 12, $\mu\, a\, (1 - e^2) = h^2$, and by equation, line 17, $e = e'$.

$$\mathrm{d}e = \frac{a}{\mu} \left\{2e - 2\cos(\upsilon - \alpha) - e \sin(\upsilon - \alpha)^2\right\} \mathrm{d}R - \frac{r^2}{\mu} \sin(\upsilon - \alpha) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \mathrm{d}\upsilon$$

$$e\, \mathrm{d}\alpha = \frac{a}{\mu} \left\{e \cos(\upsilon - \alpha) - 2\right\} \sin(\upsilon - \alpha)\, \mathrm{d}R + \frac{r^2}{\mu} \cos(\upsilon - \alpha) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right) \mathrm{d}\upsilon$$

$$\int n\, \mathrm{d}t + \varepsilon - \varpi = \upsilon - \alpha - e \sin(\upsilon - \alpha)$$

$$\mathrm{d}\varepsilon - \mathrm{d}\varpi = -\{1 - e\cos(\upsilon - \alpha)\}\, \mathrm{d}\alpha - \sin(\upsilon - \alpha)\, \mathrm{d}e$$

$$\mathrm{d}\varepsilon - \mathrm{d}\varpi = -\frac{r^2}{a^2 \sqrt{1 - e^2}}\, \mathrm{d}\varpi - \left\{\frac{r}{a(1 - e^2)} + 1\right\} \sin(\upsilon - \alpha)\, \mathrm{d}e$$

$$\frac{\frac{h^2}{\mu}}{1 + e\cos(\lambda - \varpi)} = a\{1 - e\cos(\upsilon - \alpha)\}$$

$$\mathrm{d}e = -\frac{a n\, \mathrm{d}t}{\mu \sqrt{1 - e^2}} \left\{2\cos(\lambda - \varpi) + e + e\cos(\lambda - \varpi)^2\right\} \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)$$
$$- \frac{a^2 n\, \mathrm{d}t \sqrt{1 - e^2}}{\mu} \sin(\lambda - \varpi) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$$

$$e\, \mathrm{d}\varpi = -\frac{a n\, \mathrm{d}t}{\mu \sqrt{1 - e^2}} \left\{2 + e\cos(\lambda - \varpi)\right\} \sin(\lambda - \varpi) \left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right)$$
$$+ \frac{a^2 n\, \mathrm{d}t \sqrt{1 - e^2}}{\mu} \cos(\lambda - \varpi) \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$$

$$\mathrm{d}\varepsilon - \mathrm{d}\varpi = -\sqrt{1 - e^2}\, \mathrm{d}\varpi + \frac{2a^2 n (1 - e^2)\, \mathrm{d}t}{\mu \{1 + e\cos(\lambda - \varpi)\}} \left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$$

If the longitudes be reckoned from the perihelion of the planet P,

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{a n (\cos\lambda + e)}{\mu \sqrt{1 - e^2}} \left(\frac{\mathrm{d}\mathrm{R}}{\mathrm{d}\lambda}\right) - \frac{m_{,} a^2 n \sqrt{1 - e^2}}{\mu} \left\{\frac{r_{,}' \sin\lambda_{,}'}{r_{,}^3} + \frac{r\sin\lambda - r_{,}' \sin\lambda_{,}'}{\{r^2 - 2 r r_{,}' \cos(\lambda - \lambda_{,}') + r_{,}'^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{\mathrm{d}\varpi}{\mathrm{d}t} = -\frac{a\,n\sin\lambda}{\mu\sqrt{1-e^2}}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) - \frac{m_{\prime}a^2 n\sqrt{1-e^2}}{\mu}\left\{\frac{r_{\prime}'\cos\lambda_{\prime}'}{r_{\prime}^3} + \frac{r\cos\lambda - r_{\prime}'\cos\lambda_{\prime}'}{\{r^2 - 2\,r\,r_{\prime}'\cos(\lambda-\lambda_{\prime}') + r_{\prime}^2\}^{\frac{3}{2}}}\right\}$$

and since $n\,\mathrm{d}t = \frac{r}{a}\,\mathrm{d}\,v$,

$$\frac{\mathrm{d}e}{\mathrm{d}v} = -\frac{r(\cos\lambda + e)}{\mu\sqrt{1-e^2}}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) - \frac{m_{\prime}a\,r\sqrt{1-e^2}}{\mu}\left\{\frac{r_{\prime}'\sin\lambda_{\prime}'}{r_{\prime}^3} + \frac{r\sin\lambda - r_{\prime}'\sin\lambda_{\prime}'}{\{r^2 - 2\,r\,r_{\prime}'\cos(\lambda-\lambda_{\prime}') + r_{\prime}^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{e\,\mathrm{d}\varpi}{\mathrm{d}v} = -\frac{r\sin\lambda}{\mu\sqrt{1-e^2}}\left(\frac{\mathrm{d}R}{\mathrm{d}\lambda}\right) + \frac{m_{\prime}a\,r\sqrt{1-e^2}}{\mu}\left\{\frac{r_{\prime}'\cos\lambda_{\prime}'}{r_{\prime}^3} + \frac{r\cos\lambda - r_{\prime}'\cos\lambda_{\prime}'}{\{r^2 - 2\,r\,r_{\prime}'\cos(\lambda-\lambda_{\prime}') + r_{\prime}^2\}^{\frac{3}{2}}}\right\}$$

$$\frac{\mathrm{d}\varepsilon - \mathrm{d}\varpi}{\mathrm{d}v} = -\frac{r^2}{a^2\sqrt{1-e^2}}\,\frac{\mathrm{d}\varpi}{\mathrm{d}v} - \left\{\frac{r}{a(1-e^2)} + 1\right\}\sin v\,\frac{\mathrm{d}e}{\mathrm{d}v}$$

$$\mathrm{d}a = -\frac{2a^2}{\mu}\,\mathrm{d}R$$

and since $r^2\,\mathrm{d}\lambda = \sqrt{\mu\,a\,(1-e^2)}\,\mathrm{d}t = a\sqrt{1-e^2}\,r\,\mathrm{d}v$

$$\frac{\mathrm{d}\iota}{\mathrm{d}v} + \frac{a\sqrt{1-e^2}\,r\cos\lambda}{\mu}\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}v} + \frac{a\sqrt{1-e^2}\,r\sin\lambda}{\mu}\left(\frac{\mathrm{d}R}{\mathrm{d}s}\right) = 0.$$

The last six equations serve to determine the perturbations of a comet.

Let $(\Delta\,e)_n$ be the variation of any element e during the variation $\Delta\,v$ of v at any given epoch n, neglecting the square and higher powers of $\Delta\,v$,

$$(\Delta\,e)_n = \left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_n \Delta\,v$$

If the values of $(\Delta\,e)_n$ be calculated for the epochs $0, 1, 2 \ldots m$ corresponding to the values v, $v + i\,\Delta\,v$, $v + 2\,i\,\Delta\,v$, &c., differing from each other by $i\,\Delta\,v$, then the whole variation $(\delta\,e)$ of v corresponding to the variation $i\,\Delta\,v$ of v

$$= i\left\{(\Delta\,e)_0 + (\Delta\,e)_1 \ldots\ldots\ldots + (\Delta\,e)_{m-1}\right\}$$

$$+ \frac{i-1}{2}\left\{(\Delta\,e)_m - (\Delta\,e)_0\right\} - \frac{(i-1)(i+1)}{12\,i}\left\{(\Delta^2\,e)_m - (\Delta^2\,e)_0\right\} + \text{\&c.}$$

$$= i\left\{\left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_0 + \left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_1 \ldots\ldots\ldots + \left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_{m-1}\right\}\Delta\,v$$

$$+ \frac{i-1}{2}\left\{\left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_m - \left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_0\right\}\Delta\,v - \frac{(i-1)(i+1)}{12\,i}\left\{\Delta\left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_m - \Delta\left(\frac{\mathrm{d}e}{\mathrm{d}v}\right)_0\right\}\Delta\,v.$$

When the interval $\Delta\ v$ is indefinitely diminished, $i\ \Delta\ v$ is still equal to the variation of v between the epochs for which the quantities $\left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_0$, $\left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_1$ &c. are calculated, and

$$\delta\, e = i\,\Delta\, v\left\{\left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_0 + \left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_1 \cdots\cdots + \left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_{m-1} + \frac{1}{2}\left\{\left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_m - \left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_0\right\}\right.$$
$$\left. - \frac{1}{12}\left\{\Delta\left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_m - \Delta\left(\frac{\mathrm{d}\,e}{\mathrm{d}\,v}\right)_0\right\} + \text{\&c.}\right\}$$

If the radius be taken for unity, and $i\ \Delta\ v$ is the mth part of the circumference; $i\,\Delta\, v = \frac{2\,\times\,3{\cdot}14159}{m}$, or, in other words, the resulting values of $\delta\ e$ and $\delta\ a$ in the equations given above must be multiplied by 2×3.14159, and divided by 360° expressed in the same unit as $i\ \Delta\ v$.

n is equal to the angular circumference divided by the periodic time expressed in the same unit as t; so that if a degree be taken as the unity of angular circumference, $n = 360°$ divided by the periodic time expressed in the same unit as t.

In the elliptic movement or first approximation

$\lambda = n\,t + \varepsilon +$ a series of sines of arcs multiples of $n\,t$ &c.

$\lambda_{\prime} = n_{\prime}\,t + \varepsilon_{\prime} +$ a series of sines of arcs multiples of $n_{\prime}\,t$ &c.

$\frac{a}{r} =$ constant $+$ a series of cosines.

$s =$ a series of sines.

$s_{\prime} =$ a series of sines.

These values being substituted in the equations of p. 334 give $\frac{\mathrm{d}\,e}{\mathrm{d}\,\lambda}$, $\frac{\mathrm{d}\,h}{\mathrm{d}\,\lambda}$ and $\frac{\mathrm{d}\,\iota}{\mathrm{d}\,\lambda}$ each equal to a series of sines without any constant quantity, and $\frac{\mathrm{d}\,\varpi}{\mathrm{d}\,\lambda}$, $\frac{\mathrm{d}\,\nu}{\mathrm{d}\,\lambda}$ and $\frac{\mathrm{d}\,\varepsilon}{\mathrm{d}\,\lambda}$ each equal to a series of cosines $+$ a constant quantity.

In the second approximation the values of λ, r and s retain the same form; and it is easy to see from the form of the expressions for $\frac{\mathrm{d}\,e}{\mathrm{d}\,\lambda}$, $\frac{\mathrm{d}\,h}{\mathrm{d}\,\lambda}$, &c. p. 334, that the form of the values of these quantities is not altered however far the approximation be carried.

If the sun or primary be a spheroid, ω the angle which the plane of the sun's equator makes with the plane of the orbit of the planet; and if the longitude be reckoned from the line of intersection of the sun's equator with the orbit of the planet; R is increased by the quantity $c\left\{\frac{3\sin^2\omega\sin^2\lambda - 1}{r^3}\right\}$, c being a constant dependent upon the figure of the sun; but the partial differential coefficients of this quantity, which are introduced into the values of d e, d ϖ, &c. do not change the form of the expressions for those quantities.

If the planet move in a medium which resists according to any power n of the velocity, if c be a constant and v the velocity, the term $2\,c\int v^{n+1}\,\mathrm{d}\,t$ must be added to $2\int \mathrm{d}\,R$,

$$c\,v^{n-1}\left\{(1+s^2)\frac{\mathrm{d}\,r'}{\mathrm{d}\,t} + r'\,s\,\frac{\mathrm{d}\,s}{\mathrm{d}\,t}\right\} \text{ to } \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r'}\right),$$

$$c\,v^{n-1}\,r'^2\,\frac{\mathrm{d}\,\lambda'}{\mathrm{d}\,t} \text{ to } \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda'}\right), \quad \text{and } c\,v^{n-1}\,r'\,\frac{\mathrm{d}\,.\,r'\,s}{\mathrm{d}\,t} \text{ to } \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)$$

in the equations of p. 330.

If the orbit of the planet be supposed to coincide with the plane $x\,y$, so that $s = 0$, then by the equations of p. 337 after reductions

$$\mathrm{d}\,a = -2\,c\,a^2\left(\frac{\mu}{a}\right)^{\frac{n+1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n+1}{2}}\frac{(1-e\cos v)}{n}\,\mathrm{d}\,v$$

$$\mathrm{d}\,e = -2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\left(\frac{1-e^2}{n}\right)\cos v\,\mathrm{d}\,v$$

$$e\,\mathrm{d}\,\varpi = -2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\frac{\sqrt{1-e^2}}{n}\sin v\,\mathrm{d}\,v$$

$$\mathrm{d}\,\varepsilon - \mathrm{d}\,\varpi = 2\,c\left(\frac{\mu}{a}\right)^{\frac{n-1}{2}}\left\{\frac{1+e\cos v}{1-e\cos v}\right\}^{\frac{n-1}{2}}\frac{\sin v}{n}\left\{\frac{1-e^3\cos v}{e}\right\}\mathrm{d}\,v$$

The form of these equations differs from that which obtained before, now the variations of e, ϖ and ε are periodical, while that of a has a term which varies with the time. $\frac{\mathrm{d}\,e}{\mathrm{d}\,v}$ contains only odd powers of cos v and for that reason has no constant term. The periods of the periodic inequalities of all the elliptic constants due to the action of the resisting medium are fractional parts of the periodic time of the planet.

If the origin of t coincides with the instant of the perihelion passage, by LAGRANGE's theorem

$$\cos v = \cos n t - e \sin n t^2 - \frac{e^2}{2} . \frac{\mathrm{d} . \sin n t^3}{\mathrm{d} . n t} - \frac{e^3}{2 . 3} . \frac{\mathrm{d}^2 . \sin n t^4}{(\mathrm{d} . n t)^2} - \frac{e^4}{2 . 3 . 4} \frac{\mathrm{d}^3 \sin n t^5}{(\mathrm{d} . n t^3} - \&\mathrm{c}.$$

$$\sin v = \sin n t + e \sin n t \cos n t + \frac{e^2}{2} \frac{\mathrm{d} . \sin n t^2 \cos n t}{\mathrm{d} . n t} + \frac{e^3}{2 . 3} \frac{\mathrm{d}^2 . \sin n t^3 \cos n t}{(\mathrm{d} . n t)^2} + \frac{e^4}{2 . 3 . 4} \frac{\mathrm{d}^3 . \sin n t^4 \cos n t}{(\mathrm{d} . n t)^3}$$

$$\sin n t^2 = \frac{1 - \cos 2 n t}{2}, \quad \sin n t^3 = \frac{3 \sin n t - \sin 3 n t}{4}$$

$$\sin n t^4 = \frac{3 - 4 \cos 2 n t + \cos 4 n t}{8}, \quad \sin n t^5 = \frac{10 \sin n t - 5 \sin 3 n t + \sin 5 n t}{16}$$

$$\frac{\mathrm{d} . \sin n t^3}{\mathrm{d} . n t} = \frac{3 \cos n t - 3 \cos 3 n t}{4}$$

$$\frac{\mathrm{d} . \sin n t^4}{\mathrm{d} . n t} = \frac{2 \sin 2 n t - \sin 4 n t}{2}, \quad \frac{\mathrm{d}^2 . \sin n t^4}{(\mathrm{d} . n t)^2} = 2 \cos 2 n t - 2 \cos 4 n t$$

$$\frac{\mathrm{d} . \sin n t^5}{\mathrm{d} . n t} = \frac{10 \cos n t - 15 \cos 3 n t + 5 \cos 5 n t}{16}$$

$$\frac{\mathrm{d}^2 \sin n t^5}{(\mathrm{d} . n t)^2} = \frac{- 10 \sin n t + 45 \sin 3 n t - 25 \sin 5 n t}{16}$$

$$\frac{\mathrm{d}^3 . \sin n t^5}{(\mathrm{d} . n t)} = \frac{- 10 \cos n t + 135 \cos 3 n t - 125 \cos 5 n t}{16}$$

$$\cos v = \cos n t - \frac{e}{2} + \frac{e}{2} \cos 2 n t - \frac{e^2}{2} \left\{ \frac{3 \cos n t - 3 \cos 3 n t}{4} \right\} - \frac{e^3}{2 . 3} \left\{ 2 \cos 2 n t - 2 \cos 4 n t \right\}$$
$$- \frac{e^4}{2 . 3 . 4} \left\{ \frac{- 10 \cos n t + 135 \cos 3 n t - 125 \cos 5 n t}{16} \right\}$$

If the origin of the time does not coincide with the perihelion passage, $n t + \varepsilon - \varpi$ must be substituted for $n t$, but as ε always accompanies $n t$, it may be suppressed at present for convenience, and afterwards replaced.

$$\cos v = \left\{ 1 - \frac{3}{8} e^2 + \frac{5 e^4}{192} \right\} \cos (n t - \varpi) - \frac{e}{2} + \frac{e}{2} \left\{ 1 - \frac{2 e^2}{3} \right\} \cos (2 n t - 2 \varpi)$$
$$+ \frac{3}{8} e^2 \left\{ 1 - \frac{15}{16} e^2 \right\} \cos (3 n t - 3 \varpi) + \frac{e^3}{3} \cos (4 n t - 4 \varpi) + \frac{125}{384} e^4 \cos (5 n t - 5 \varpi)$$

$$\sin n t^2 \cos n t = \frac{\mathrm{d} . \sin n t^3}{3 \mathrm{d} . n t} = \frac{\cos n t - \cos 3 n t}{4}, \quad \frac{\mathrm{d} . \sin n t^2 \cos n t}{\mathrm{d} . n t} = \frac{- \sin n t + 3 \sin 3 n t}{4}$$

$$\sin n\,t^3 \cos n\,t = \frac{\mathrm{d}\,.\sin n\,t^4}{4\,\mathrm{d}\,.\,n\,t} = \frac{2\sin 2\,n\,t - \sin 4\,n\,t}{8}, \qquad \frac{\mathrm{d}\,.\sin n\,t^3 \cos n\,t}{\mathrm{d}\,.\,n\,t} = \frac{\cos 2\,n\,t - \cos 4\,n\,t}{2}$$

$$\frac{\mathrm{d}^2\,.\sin n\,t^3 \cos n\,t}{(\mathrm{d}\,.\,n\,t)^2} = -\sin 2\,n\,t + 2\sin 4\,n\,t$$

$$\sin n\,t^4 \cos n\,t = \frac{\mathrm{d}\,.\sin n\,t^5}{5\,\mathrm{d}\,.\,n\,t} = \frac{2\cos n\,t - 3\cos 3\,n\,t + \cos 5\,n\,t}{16}$$

$$\frac{\mathrm{d}\,.\sin n\,t^4 \cos n\,t}{\mathrm{d}\,.\,n\,t} = \frac{-2\sin n\,t + 9\sin 3\,n\,t - 5\sin 5\,n\,t}{16}$$

$$\frac{\mathrm{d}^2\,.\sin n\,t^4 \cos n\,t}{(\mathrm{d}\,.\,n\,t)^2} = \frac{-2\cos n\,t + 27\cos 3\,n\,t - 25\cos 5\,n\,t}{16}$$

$$\frac{\mathrm{d}^3\,.\sin n\,t^4 \cos n\,t}{(\mathrm{d}\,.\,n\,t)^3} = \frac{2\sin n\,t - 81\sin 3\,n\,t + 125\sin 5\,n\,t}{16}$$

$$\sin \upsilon = \sin n t + \frac{e}{2}\sin 2\,n t + \frac{e^2}{2}\left\{\frac{-\sin n\,t + 3\sin 3\,n\,t}{4}\right\} + \frac{e^3}{2\,.\,3}\{-\sin 2\,n\,t + 2\sin 4 n t\}$$
$$+ \frac{e^4}{2\,.\,3\,.\,4}\left\{\frac{2\sin n t - 81\sin 3\,n t + 125\sin 5\,n t}{16}\right\}$$

and replacing $n\,t$ by $n\,t - \varpi$

$$\sin \upsilon = \left\{1 - \frac{3\,e^2}{8} + \frac{5}{192}\,e^4\right\}\sin(n\,t - \varpi) + \frac{e}{2}\left\{1 - 2\,\frac{e^2}{3}\right\}\sin(2\,n\,t - 2\,\varpi)$$
$$+ \left\{\frac{2\,e^2}{8} - \frac{4}{192}\,e^4\right\}\sin(n\,t - \varpi) \qquad + \frac{e^3}{6}\ \sin(2\,n\,t - 2\,\varpi)$$
$$+ \frac{3\,e^2}{8}\left\{1 - \frac{15}{16}\,e^2\right\}\sin(3\,n\,t - 3\,\varpi) + \frac{e^3}{3}\sin(4\,n\,t - 4\,\varpi) + \frac{125}{384}\,e^4\sin(5 n t - 5\,\varpi)$$
$$+ \frac{3}{8}\cdot\frac{6}{16}\,e^4\sin(3\,n\,t - 3\,\varpi)$$

$$\cos(\upsilon + \varpi) = \left\{1 - \frac{e^2}{4} + \frac{e^4}{64}\right\}\cos n\,t - \frac{e}{2}\cos\varpi + \frac{e}{2}\left\{1 - \frac{e^2}{2}\right\}\cos(2 n t - \varpi)$$
$$+ \frac{3}{8}\,e^2\left\{1 - \frac{3}{4}\,e^2\right\}\cos(3\,n\,t - 2\,\varpi) + \frac{e^3}{3}\cos(4\,n\,t - 3\,\varpi) + \frac{125}{384}\,e^4\cos(5\,n t - 4\,\varpi)$$
$$- \frac{e^2}{8}\left\{1 - \frac{e^2}{12}\right\}\cos(n\,t - 2\,\varpi) - \frac{e^3}{12}\cos(2\,n\,t - 3\,\varpi) - \frac{9}{128}\,e^4\cos(3\,n\,t - 4\,\varpi)$$

$$\sin(\upsilon + \varpi) = \left\{1 - \frac{e^2}{4} + \frac{e^4}{64}\right\}\sin n\,t - \frac{e}{2}\sin\varpi + \frac{e}{2}\left\{1 - \frac{e^2}{2}\right\}\sin(2 n t - \varpi)$$
$$+ \frac{3}{8}\,e^2\left\{1 - \frac{3}{4}\,e^2\right\}\sin(3\,n\,t - 2\,\varpi) + \frac{e^3}{3}\sin(4\,n\,t - 3\,\varpi) + \frac{125}{384}\,e^4\sin(5\,n\,t - 4\,\varpi)$$

$$+\frac{e^2}{8}\left(1-\frac{e^2}{12}\right)\sin(nt-2\varpi)+\frac{e^3}{12}\sin(2nt-3\varpi)+\frac{9}{128}e^4\sin(3nt-4\varpi)$$

$$\cos(\upsilon-\varpi)=\left(1-\frac{e^2}{4}\right)\cos(nt-2\varpi)-\frac{e}{2}\cos\varpi+\frac{e}{2}\cos(2nt-3\varpi)+\frac{3}{8}e^2\cos(3nt-4\varpi)-\frac{e^2}{8}\cos nt$$

$$\sin(\upsilon-\varpi)=\left(1-\frac{e^2}{4}\right)\sin(nt-2\varpi)+\frac{e}{2}\sin\varpi+\frac{e}{2}\sin(2nt-3\varpi)+\frac{3}{8}e^2\sin(3nt-4\varpi)+\frac{e^2}{8}\sin nt$$

$$r\cos\lambda=r\cos(\lambda-\varpi)\cos\varpi-r\sin(\lambda-\varpi)\sin\varpi=a\{(\cos\upsilon-e)\cos\varpi-(1-e^2)^{\frac{1}{2}}\sin\upsilon\sin\varpi\}$$

$$r\sin\lambda=r\sin(\lambda-\varpi)\cos\varpi+r\cos(\lambda-\varpi)\sin\varpi=a\{(1-e^2)^{\frac{1}{2}}\sin\upsilon\cos\varpi+(\cos\upsilon-e)\sin\varpi\}$$

$$r\cos\lambda=a\left\{\left(1-\frac{e^2}{4}-\frac{e^4}{16}\right)\cos(\upsilon+\varpi)-e\cos\varpi+\frac{e^2}{4}\left(1+\frac{e^2}{4}\right)\cos(\upsilon-\varpi)\right\}$$

$$r\sin\lambda=a\left\{\left(1-\frac{e^2}{4}-\frac{e^4}{16}\right)\sin(\upsilon+\varpi)-e\sin\varpi-\frac{e^2}{4}\left(1+\frac{e^2}{4}\right)\sin(\upsilon-\varpi)\right\}$$

$$\genfrac{}{}{0pt}{}{r\cos\lambda}{r\sin\lambda}=a\left\{\left(1-\frac{e^2}{2}-\frac{e^4}{64}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}nt-\frac{3e}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}\varpi+\frac{e}{2}\left(1-\frac{3}{4}e^2\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi)\right.$$

$$+\frac{3}{8}e^2\left(1-e^2\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-2\varpi)+\frac{e^3}{3}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-3\varpi)+\frac{125}{384}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(5nt-4\varpi)$$

$$\left.\pm\frac{e^2}{8}\left(1+\frac{e^2}{3}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-2\varpi)\pm\frac{e^3}{24}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-3\varpi)\pm\frac{3}{128}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-4\varpi)\text{ \&c.}\right\}$$

$$rr_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}\{\lambda-\lambda_{\prime}\}=aa_{\prime}\left\{\left(1-\frac{e^2}{2}-\frac{e^4}{64}\right)\left(1-\frac{e_{\prime}^2}{2}-\frac{e_{\prime}^4}{64}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{\prime}t)\right.$$

$$\mp\frac{3}{2}e\left(1-\frac{e_{\prime}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{\prime}t-\varpi)+\frac{e}{2}\left(1-\frac{3}{4}e^2\right)\left(1-\frac{e_{\prime}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-n_{\prime}t-\varpi)$$

$$+\frac{3}{8}e^2(1-e^2)\left(1-\frac{e_{\prime}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-n_{\prime}t-2\varpi)+\frac{e^3}{3}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-n_{\prime}t-3\varpi)$$

$$+\frac{125}{384}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(5nt-n_{\prime}t-4\varpi)\pm\frac{e^2}{8}\left(1+\frac{e^2}{3}\right)\left(1-\frac{e_{\prime}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{\prime}t-2\varpi)$$

$$\pm\frac{e^3}{24}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{\prime}t-3\varpi)\pm\frac{3}{128}e^4\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{\prime}t-4\varpi)$$

$$-\frac{3}{2}e_{\prime}\left(1-\frac{e^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-\varpi_{\prime})+\frac{9}{4}ee_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(\varpi-\varpi_{\prime})$$

$$-\frac{3}{4}e\,e_{\prime}\left(1-\frac{3}{4}e^2\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi-\varpi_{\prime})-\frac{9}{16}e^2e_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-2\varpi-\varpi_{\prime})$$

$$-\frac{e^3e_{\prime}}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-3\varpi-\varpi_{\prime})\mp\frac{3}{16}e^2e_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-2\varpi+\varpi_{\prime})\mp\frac{e^3e_{\prime}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-3\varpi+\varpi_{\prime})$$

$$+\frac{e_{,}}{2}\left(1-\frac{3}{4}e_{,}^{2}\right)\left(1-\frac{e^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-2n_{,}t+\varpi_{,})\mp\frac{3}{4}ee_{,}\left(1-\frac{3}{4}e_{,}^{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2n_{,}t-\varpi-\varpi_{,})$$

$$+\frac{ee_{,}}{4}\left(1-\frac{3}{4}e^{2}\right)\left(1-\frac{3}{4}e_{,}^{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-2n_{,}t-\varpi+\varpi_{,})+\frac{3}{16}e^{2}e_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-2n_{,}t-2\varpi+\varpi_{,})$$

$$+\frac{e^{3}e_{,}}{6}\genfrac{}{}{0pt}{}{\cos}{\sin}(4nt-2n_{,}t-3\varpi+\varpi_{,})\pm\frac{e^{2}e_{,}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+2n_{,}t-2\varpi-\varpi_{,})$$

$$\pm\frac{e^{3}e_{,}}{48}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+2n_{,}t-3\varpi-\varpi_{,})+\frac{3}{8}e_{,}^{2}(1-e_{,}^{2})\left(1-\frac{e^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-3n_{,}t+2\varpi_{,})$$

$$\mp\frac{9}{16}ee_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3n_{,}t-\varpi-2\varpi_{,})+\frac{3}{16}ee_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-3n_{,}t-\varpi+2\varpi_{,})$$

$$+\frac{9}{64}e^{2}e_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt-3n_{,}t-2\varpi+2\varpi_{,})\pm\frac{3}{64}e^{2}e_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+3n_{,}t-2\varpi-2\varpi_{,})$$

$$+\frac{e^{3}}{3}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-4n_{,}t+3\varpi_{,})\mp\frac{e\,e_{,}^{3}}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(4n_{,}t-\varpi-3\varpi_{,})$$

$$+\frac{e\,e_{,}^{3}}{6}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-4n_{,}t-\varpi+3\varpi_{,})+\frac{125}{384}e_{,}^{4}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-5n_{,}t+4\varpi_{,})$$

$$+\frac{e_{,}^{2}}{8}\left(1+\frac{e^{2}}{3}e_{,}^{2}\right)\left(1-\frac{e^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{,}t-2\varpi_{,})-\frac{3}{16}ee_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{,}t+\varpi-2\varpi_{,})$$

$$+\frac{e\,e_{,}^{2}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{,}t-\varpi-2\varpi_{,})+\frac{3}{64}e^{2}e_{,}^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{,}t-2\varpi-2\varpi_{,})$$

$$\pm\frac{e^{2}e_{,}^{2}}{64}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{,}t-2\varpi+2\varpi_{,})+\frac{e_{,}^{3}}{24}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+2n_{,}t-3\varpi_{,})$$

$$\mp\frac{e\,e_{,}^{3}}{16}\genfrac{}{}{0pt}{}{\cos}{\sin}(2n_{,}t+\varpi-3\varpi_{,})+\frac{e\,e_{,}^{3}}{48}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+2n_{,}t-\varpi-3\varpi_{,})$$

$$+\frac{}{128}e_{,}^{4}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+3n_{,}t-4\varpi_{,})+\&c.$$

$$rr_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}\{\lambda+\lambda_{,}\}=aa_{,}\left\{\left(1-\frac{e^{2}+e_{,}^{2}}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{,}t)\right.$$

$$-\frac{3}{2}e\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{,}t+\varpi)+\frac{e}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{,}t-\varpi)$$

$$+\frac{3}{8}e^{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{,}t-2\varpi)\pm\frac{e^{2}}{8}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{,}t-2\varpi)$$

$$-\frac{3}{2}e_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+\varpi_{,})+\frac{9}{4}ee_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(\varpi+\varpi_{,})-\frac{3}{4}ee_{,}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi+\varpi_{,})$$

$$+\frac{e_{\prime}}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+2n_{\prime}t-\varpi_{\prime})-\frac{3}{4}ee_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(2n_{\prime}t+\varpi-\varpi_{\prime})+\frac{ee_{\prime}}{4}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+2nt-\varpi-\varpi_{\prime})+\&c.$$

$$rr_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}\{\lambda+\lambda_{\prime}-2\nu\}=aa_{\prime}\left\{\left(1-\frac{e^2+e_{\prime}^2}{2}\right)\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+n_{\prime}t-2\nu)\right.$$

$$-\frac{3}{2}e\genfrac{}{}{0pt}{}{\cos}{\sin}(n_{\prime}t+\varpi-2\nu)+\frac{e}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+n_{\prime}t-\varpi-2\nu)$$

$$+\frac{3}{8}e^2\genfrac{}{}{0pt}{}{\cos}{\sin}(3nt+n_{\prime}t-2\varpi-2\nu)\pm\frac{e^2}{8}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt-n_{\prime}t-2\varpi+2\nu)$$

$$-\frac{3}{2}e_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+\varpi_{\prime}-2\nu)+\frac{9}{4}e\,e_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(\varpi+\varpi_{\prime}-2\nu)$$

$$-\frac{3}{4}ee_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt-\varpi+\varpi_{\prime}-2\nu)+\frac{e_{\prime}}{2}\genfrac{}{}{0pt}{}{\cos}{\sin}(nt+2n_{\prime}t-\varpi_{\prime}-2\nu)$$

$$-\frac{3}{4}ee_{\prime}\genfrac{}{}{0pt}{}{\cos}{\sin}(2n_{\prime}t+\varpi-\varpi_{\prime}-2\nu)+\frac{ee_{\prime}}{4}\genfrac{}{}{0pt}{}{\cos}{\sin}(2nt+2n_{\prime}t-\varpi-\varpi_{\prime}-2\nu)+\&c.$$

$$r=a\left\{1+\frac{e^2}{2}-e\left(1-\frac{3}{8}e^2\right)\cos(nt-\varpi)-\frac{e^2}{2}\left(1-\frac{2e^2}{3}\right)\cos(2nt-2\varpi)\right.$$

$$-\frac{3}{8}e^3\cos(3nt-3\varpi)-\frac{e^4}{3}\cos(4nt-4\varpi)+\&c.$$

$$r^2=a^2\left\{1+\frac{3e^2}{2}-2e\left(1-\frac{e^2}{8}\right)\cos(nt-\varpi)-\frac{e^2}{2}\left(1-\frac{e^2}{3}\right)\cos(2nt-2\varpi)\right.$$

$$-\frac{e^3}{4}\cos(3nt-3\varpi)-\frac{e^4}{6}\cos(4nt-4\varpi)+\&c.$$

$$r^{-3}=a^{-3}\left\{1+\frac{3}{2}e^2\left(1+\frac{e^2}{2}\right)+3e\left(1+\frac{9}{8}e^2\right)\cos(nt-\varpi)\right.$$

$$+\frac{9}{2}e^2\left(1-\frac{e^2}{18}\right)\cos 2nt-2\varpi+\frac{53}{8}e^3\cos(3nt-3\varpi)$$

$$+\frac{31}{4}e^4\cos(4nt-4\varpi)+\&c.$$

If the coefficient of $\cos n\,\theta$ in the development of $\left\{1-\frac{2a}{a_{\prime}}\cos\theta+\frac{a^2}{a_{\prime}^2}\right\}^{-m}$ according to cosines of arcs mulitples of θ is called $b_{2m,n}$

$$b_{1,0}=1+\left(\frac{1}{2}\right)^2\frac{a^2}{a_{\prime}^2}+\left(\frac{1.3}{2.4}\right)^2\frac{a^4}{a_{\prime}^4}+\left(\frac{1.3.5}{2.4.6}\right)^2\frac{a^6}{a_{\prime}^6}+\&c.$$

$$b_{1,1} = \frac{a}{a_{,}} + \frac{1.3}{2.4}\frac{a^3}{a_{,}^3} + \frac{1.3.3.5}{2.4.4.6}\frac{a^5}{a_{,}^5} + \&c.$$

$$b_{1,2} = \frac{3}{4}\frac{a^2}{a_{,}^2} + \frac{1.3.5}{2.4.6}\frac{a^4}{a_{,}^4} + \frac{1.3.3.5.7}{2.4.4.6.8}\frac{a^6}{a_{,}^6} + \&c.$$

$$b_{1,3} = \frac{3.5}{4.6}\frac{a^3}{a_{,}^3} + \frac{1.3.5.7}{2.4.6.8}\frac{a^5}{a_{,}^5} + \frac{1.3.3.5.7.9}{2.4.4.6.8.10}\frac{a^7}{a_{,}^7} + \&c.$$

$$b_{3,0} = 1 + \left(\frac{3}{2}\right)^2\frac{a^2}{a_{,}^2} + \left(\frac{3.5}{2.4}\right)^2\frac{a^4}{a_{,}^4} + \left(\frac{3.5.7}{2.4.6}\right)^2\frac{a^6}{a_{,}^6} + \&c.$$

$$b_{3,1} = \frac{3a}{a_{,}} + \frac{3.3.5}{2.4}\frac{a^3}{a_{,}^3} + \frac{3.5.3.5.7}{2.4.4.6}\frac{a^5}{a_{,}^5} + \&c.$$

$$b_{3,2} = \frac{3.5}{4}\frac{a^2}{a_{,}^2} + \frac{3.3.5.7}{2.4.6}\frac{a^4}{a_{,}^4} + \frac{3.5.3.5.7}{2.4.4.6.8}\frac{a^6}{a_{,}^6} + \&c.$$

$$b_{3,3} = \frac{3.5.7}{4.6}\frac{a^3}{a_{,}^3} + \frac{3.3.5.7.9}{2.4.6.8}\frac{a^5}{a_{,}^5} + \frac{3.5.3.5.7.9.11}{2.4.4.6.8.10}\frac{a^7}{a_{,}^7} + \&c.$$

$$b_{5,0} = 1 + \left(\frac{5}{2}\right)^2\frac{a^2}{a_{,}^2} + \left(\frac{5.7}{2.4}\right)^2\frac{a^4}{a_{,}^4} + \left(\frac{5.7.9}{2.4.6}\right)^2\frac{a^6}{a_{,}^6} + \&c.$$

$$b_{5,1} = 5\frac{a}{a_{,}} + \frac{5.5.7}{2.4}\frac{a^3}{a_{,}^3} + \frac{5.7.5.7.9}{2.4.4.6}\frac{a^5}{a_{,}^5} + \&c.$$

$$b_{5,2} = \frac{5.7}{4}\frac{a^2}{a_{,}^2} + \frac{5.5.7.9}{2.4.6}\frac{a^4}{a_{,}^4} + \frac{5.7.5.7.9.11}{2.4.4.6.8}\frac{a^6}{a_{,}^6} + \&c.$$

$$b_{5,3} = \frac{5.7.9}{4.6}\frac{a^3}{a_{,}^3} + \frac{5.5.7.9.11}{2.4.6.8}\frac{a^5}{a_{,}^5} + \frac{5.7.5.7.9.11.13}{2.4.4.6.8.10}\frac{a^7}{a_{,}^7}$$

$$b_{3,1} = 3\frac{a}{a_{,}}\left\{b_{5,0} - \frac{1}{2}b_{5,2}\right\} \qquad b_{1,1} = \frac{a}{a_{,}}\left\{b_{3,0} - \frac{1}{2}b_{3,2}\right\}$$

$$2\,b_{3,2} = \frac{3}{2}\frac{a}{a_{,}}\left\{b_{5,1} - b_{5,3}\right\} \qquad 2\,b_{1,2} = \frac{1}{2}\frac{a}{a_{,}}\left\{b_{3,1} - b_{3,3}\right\}$$

$$3\,b_{3,3} = \frac{3}{2}\frac{a}{a_{,}}\left\{b_{5,2} - b_{5,4}\right\} \qquad 3\,b_{1,3} = \frac{1}{2}\frac{a}{a_{,}}\left\{b_{3,2} - b_{3,4}\right\}$$

$$R = m_{,}\left\{\frac{r r_{,}' \cos(\lambda - \lambda_{,}')}{r_{,}^3} - \frac{1}{\{r^2 - 2 r r_{,}' \cos(\lambda - \lambda_{,}') + r_{,}^2\}^{\frac{1}{2}}}\right\}$$

and since

$$r r_{,}' \cos(\lambda - \lambda_{,}') = r r_{,}\left\{\cos^2\frac{\iota_{,}}{2}\cos(\lambda - \lambda_{,}) + \sin^2\frac{\iota_{,}}{2}\cos(\lambda + \lambda_{,} - 2\nu_{,})\right\}, \text{ p. } 330;$$

$$R = m_{,} \left\{ \frac{r r_{,} \left\{ \cos^2 \frac{\iota_{,}}{2} \cos(\lambda - \lambda_{,}) + \sin^2 \frac{\iota_{,}}{2} \cos(\lambda + \lambda_{,} - 2\nu_{,}) \right\}}{r_{,}^3} \right.$$

$$\left. - \frac{1}{\left\{ r^2 - 2 r r_{,} \left\{ \cos^2 \frac{\iota_{,}}{2} \cos(\lambda - \lambda_{,}) + \sin^2 \frac{\iota_{,}}{2} \cos(\lambda + \lambda_{,} - 2\nu_{,}) \right\} + r_{,}^2 \right\}^{\frac{1}{2}}} \right\}$$

Neglecting the cubes and higher powers of e

$$R = m_{,} \frac{a}{a_{,}^2} \Big\{ \Big(1 - \sin^2 \frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2}\Big) \cos(nt - n_{,}t) - \frac{3}{2} e \cos(n_{,}t - \varpi) - \frac{3}{2} e_{,} \cos(nt - \varpi_{,})$$

$$+ \frac{e}{2} \cos(2nt - n_{,}t - \varpi) + \frac{e_{,}}{2} \cos(nt - 2n_{,}t + \varpi_{,}) + \frac{3}{8} e^2 \cos(3nt - n_{,}t - 2\varpi)$$

$$+ \frac{3}{8} e_{,}^2 \cos(nt - 3n_{,}t + 2\varpi_{,}) + \frac{e^2}{8} \cos(nt + n_{,}t - 2\varpi) + \frac{e_{,}^2}{8} \cos(nt + n_{,}t - 2\varpi_{,})$$

$$+ \frac{9}{4} ee_{,} \cos(\varpi - \varpi_{,}) - \frac{3}{4} ee_{,} \cos(2nt - \varpi - \varpi_{,}) - \frac{3}{4} ee_{,} \cos(2n_{,}t - \varpi - \varpi_{,})$$

$$+ \frac{e e_{,}}{4} \cos(2nt - 2n_{,}t - \varpi + \varpi_{,}) + \sin^2 \frac{\iota}{2} \cos(nt + n_{,}t - 2\nu) \Big\} \Big\{ 1 + \frac{3}{2} e^2_{,}$$

$$+ 3 e_{,} \cos(n_{,}t - \varpi_{,}) + \frac{9}{2} e_{,}^2 \cos(2n_{,}t - 2\varpi_{,}) + \&c. \Big\}$$

$$- m_{,} \Big\{ a^2 \Big\{ 1 + \frac{3}{2} e^2 - 2 e \cos(nt - \varpi) - \frac{e^2}{2} \cos(2nt - 2\varpi) \Big\}$$

$$- 2 a a_{,} \Big\{ \Big(1 - \sin^2 \frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2}\Big) \cos(nt - n_{,}t) - \frac{3}{2} e \cos(n_{,}t - \varpi)$$

$$- \frac{3}{2} e_{,} \cos(nt - \varpi_{,}) + \frac{e}{2} \cos(2nt - n_{,}t - \varpi) + \frac{e_{,}}{2} \cos(nt - 2n_{,}t + \varpi_{,})$$

$$+ \frac{3}{8} e^2 \cos(3nt - n_{,}t - 2\varpi) + \frac{3}{8} e_{,}^2 \cos(nt - 3n_{,}t + 2\varpi_{,}) + \frac{e^2}{8} \cos(nt + n_{,}t - 2\varpi)$$

$$+ \frac{e_{,}^2}{8} \cos(nt + n_{,}t - 2\varpi_{,}) + \frac{9}{4} ee_{,} \cos(\varpi - \varpi_{,}) - \frac{3}{4} ee_{,} \cos(2nt - \varpi - \varpi_{,})$$

$$- \frac{3}{4} ee_{,} \cos(2n_{,}t - \varpi - \varpi_{,}) + \frac{e e_{,}}{4} \cos(2nt - 2n_{,}t - \varpi + \varpi_{,})$$

$$+ \sin^2 \frac{\iota}{2} \cos(nt + n_{,}t - 2\nu) \Big\} + a_{,}^2 \Big\{ 1 + \frac{3}{2} e_{,}^2$$

$$- 2 e_{,} \cos(n_{,}t - \varpi_{,}) - \frac{e_{,}^2}{2} \cos(2n_{,}t - 2\varpi_{,}) + \&c. \Big\} \Big\}^{\frac{1}{2}}$$

$= \text{terms independent of } b$

$$- \frac{m_{,}}{a_{,}} \left\{ b_{1,0} + b_{1,1} \cos (nt - n_{,}t) + b_{1,2} \cos (2nt - 2n_{,}t) + \&c. \right\}$$

$$+ \frac{m_{,}}{2 a_{,}^{3}} \left\{ b_{3,0} + b_{3,1} \cos (nt - n_{,}t) + b_{3,2} \cos (2nt - 2n_{,}t) \ \&c. \right\} \left\{ a^2 \left\{ \frac{3}{2} e^2 \right. \right.$$

$$\left. - 2e \cos (nt - \varpi) - \frac{e^2}{2} \cos (2nt - 2\varpi) \right\} - 2 a a_{,} \left\{ \left(- \sin^2 \frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2} \right) \cos (nt - n_{,}t) \right.$$

$$- \frac{3}{2} e \cos (n_{,}t - \varpi) - \frac{3}{2} e_{,} \cos (nt - \varpi_{,}) + \frac{e}{2} \cos (2nt - n_{,}t - \varpi)$$

$$+ \frac{e_{,}}{2} \cos (nt - 2n_{,}t + \varpi_{,}) + \frac{3}{8} e^2 \cos (3nt - n_{,}t - 2\varpi) + \frac{3}{8} e_{,}^2 \cos (nt - 3n_{,}t + 2\varpi_{,})$$

$$+ \frac{e^2}{8} \cos (nt + n_{,}t - 2\varpi) + \frac{e_{,}^2}{8} \cos (nt + n_{,}t - 2\varpi_{,}) + \frac{9}{4} e e_{,} \cos (\varpi - \varpi_{,})$$

$$- \frac{3}{4} e e_{,} \cos (2nt - \varpi - \varpi_{,}) - \frac{3}{4} e e_{,} \cos (2n_{,}t - \varpi - \varpi_{,}) + \frac{e e_{,}}{4} \cos (2nt - 2n_{,}t - \varpi + \varpi_{,})$$

$$\left. \left. + \sin^2 \frac{\iota_{,}}{2} \cos (nt + n_{,}t - 2\nu_{,}) \right\} + a_{,}^2 \left\{ \frac{3}{2} e_{,}^2 - 2 e_{,} \cos (n_{,}t - \varpi_{,}) - \frac{e_{,}^2}{2} \cos (2n_{,}t - 2\varpi_{,}) \&c. \right\} \right\}$$

$$- \frac{1 . 3 m_{,}}{2 . 4 a_{,}^{5}} \left\{ b_{5,0} + b_{5,1} \cos (nt - n_{,}t) + b_{5,2} \cos (2nt - 2n_{,}t) + \&c. \right\} \left\{ 2 a^2 e \cos (nt - \varpi) \right.$$

$$- 3 a a_{,} e \cos (n_{,}t - \varpi) - 3 a a_{,} e_{,} \cos (nt - \varpi_{,}) + a a_{,} e \cos (2nt - n_{,}t - \varpi)$$

$$\left. + a a_{,} e_{,} \cos (nt - 2n_{,}t + \varpi_{,}) + 2 a_{,}^2 e_{,} \cos (n_{,}t - \varpi_{,}) \right\}^2 + \&c.$$

$$\left\{ 2 a^2 e \cos (nt - \varpi) - 3 a a_{,} e \cos (n_{,}t - \varpi) - 3 a a_{,} e_{,} \cos (nt - \varpi_{,}) \right.$$

$$\left. + a a_{,} e \cos (2nt - n_{,}t - \varpi) + a a_{,} e_{,} \cos (nt - 2n_{,}t + \varpi_{,}) + 2 a_{,}^2 e_{,} \cos (n_{,}t - \varpi_{,}) \right\}^2$$

$$= 2 a^4 e^2 + 5 a^2 a_{,}^2 (e^2 + e_{,}^2) + 2 a_{,}^4 e_{,}^2 + a^2 (2 a^2 - 3 a_{,}^2) e^2 \cos (2nt - 2\varpi)$$

$$+ \frac{9}{2} a^2 a_{,}^2 e^2 \cos (2n_{,}t - 2\varpi) + \frac{9}{2} a^2 a_{,}^2 e_{,}^2 \cos (2nt - 2\varpi_{,}) + \frac{a^2 a_{,}^2}{2} e^2 \cos (4nt - 2n_{,}t - 2\varpi)$$

$$+ \frac{a^2 a_{,}^2}{2} e_{,}^2 \cos (2nt - 4n_{,}t + 2\varpi_{,}) + a_{,}^2 (2 a_{,}^2 - 3 a^2) e_{,}^2 \cos (2n_{,}t - 2\varpi_{,})$$

$$- 4 (a^2 e^2 + a_{,}^2 e_{,}^2) a a_{,} \cos (nt - n_{,}t) - 6 a^3 a_{,} e^2 \cos (nt + n_{,}t - 2\varpi)$$

$$- 6\, a a_{,}^3 e_{,}^2 \cos(nt + n_{,}t - 2\varpi_{,}) - 6\,(a^2 + a_{,}^2)\, a a_{,} e e_{,} \cos(\varpi - \varpi_{,})$$

$$- 2(3a^2 - a_{,}^2)\, a a_{,} e e_{,} \cos(2nt - \varpi - \varpi_{,}) - 2(3a_{,}^2 - a^2)\, a a_{,} e e_{,} \cos(2n_{,}t - \varpi - \varpi_{,})$$

$$+ 2\, a^3 a_{,} e^2 \cos(3nt - n_{,}t - 2\varpi) + 2\, a a_{,}^3 e_{,}^2 \cos(nt - 3n_{,}t + 2\varpi_{,})$$

$$+ 2(a^2 + a_{,}^2)\, a a_{,} e e_{,} \cos(2nt - 2n_{,}t - \varpi + \varpi_{,}) + 9\, a^2 a_{,}^2 e e_{,} \cos(nt - n_{,}t + \varpi - \varpi_{,})$$

$$+ 14\, a^2 a_{,}^2 e e_{,} \cos(nt + n_{,}t - \varpi - \varpi_{,}) - 3\, a^2 a_{,}^2 (e^2 + e_{,}^2) \cos(2nt - 2n_{,}t)$$

$$- 3\, a^2 a_{,}^2 e e_{,} \cos(nt - 3n_{,}t + \varpi + \varpi_{,}) - 3\, a^2 a_{,}^2 e e_{,} \cos(3nt - n_{,}t - \varpi - \varpi_{,})$$

$$- 2\, a^2 a_{,}^2 e e_{,} \cos(nt - n_{,}t - \varpi + \varpi_{,}) + a^2 a_{,}^2 e e_{,} \cos(3nt - 3n_{,}t - \varpi + \varpi_{,})$$

Replacing ε which accompanies nt,

$$R = m_{,} \left\{ -\frac{b_{1,0}}{a_{,}} + 3\,\frac{(a^2 e^2 + a_{,}^2 e_{,}^2)}{2\,.\,2\,a_{,}^3} b_{3,0} + \frac{a a_{,}}{2\,a_{,}^3} \left(\sin^2 \frac{\iota_{,}}{2} + \frac{e^2 + e_{,}^2}{2} \right) b_{3,1} \right.$$

$$- \frac{1\,.\,3}{2\,.\,4\,.\,a_{,}^5} \left(2a^4 e^2 + 5a^2 a_{,}^2 (e^2 + e_{,}^2) + 2a_{,}^4 e_{,}^2 \right) b_{5,0} + \frac{1\,.\,3\,.\,2}{2\,.\,4\,a_{,}^5} (a^2 e^2 + a_{,}^2 e_{,}^2)\, a a_{,} b_{5,1}$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2} \frac{a^2 a_{,}^2}{a_{,}^5} (e^2 + e_{,}^2) b_{5,2} \qquad [0^*]$$

$$+ m_{,} \left\{ \frac{a}{a_{,}^2} \left\{ \cos^2 \frac{\iota_{,}}{2} - \frac{e^2 + e_{,}^2}{2} \right\} - \frac{b_{1,1}}{a_{,}} + \frac{a a_{,}}{a_{,}^3} \left\{ \sin^2 \frac{\iota_{,}}{2} + \frac{e^2 + e_{,}^2}{2} \right\} \left\{ b_{3,0} + \tfrac{1}{2} b_{3,2} \right\} \right.$$

$$+ \frac{3(a^2 e^2 + a_{,}^2 e_{,}^2)}{2\,.\,2\,a_{,}^3} b_{3,1} + \frac{1\,.\,3\,.\,4}{2\,.\,4\,a_{,}^5} (a^2 e^2 + a_{,}^2 e_{,}^2)\, a a_{,} b_{5,0}$$

$$- \frac{1\,.\,3}{2\,.\,4\,a_{,}^5} \left\{ 2a^4 e^2 + \frac{7}{2} a^2 a_{,}^2 (e^2 + e_{,}) + 2a_{,}^4 e_{,}^2 \right\} b_{5,1} + \frac{1\,.\,3\,.\,2}{2\,.\,4\,a_{,}^5} (a^2 e^2 + a_{,}^2 e_{,}^2)\, a a_{,} b_{5,2}$$

$$\left. + \frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2} \frac{a^2 a_{,}^2}{a_{,}^5} (e^2 + e_{,}^2) b_{5,3} \right\} \cos(nt - n_{,}t + \varepsilon - \varepsilon_{,}) \qquad [1]$$

$$+ m_{,} \left\{ -\frac{b_{1,2}}{a_{,}} + \frac{a a_{,}}{2\,a_{,}^3} \left\{ \sin^2 \frac{\iota_{,}}{2} + \frac{e^2 + e_{,}^2}{2} \right\} \left\{ b_{3,1} + b_{3,3} \right\} + \frac{3(a^2 e^2 + a_{,}^2 e_{,}^2)}{2\,.\,2a_{,}^3} b_{3,2} \right.$$

$$+ \frac{1\,.\,3\,.\,3}{2\,.\,4} \frac{a^2 a_{,}^2 (e^2 + e_{,}^2)}{a_{,}^5} b_{5,0} + \frac{1\,.\,3\,.\,2}{2\,.\,4} \frac{(a^2 e^2 + a_{,}^2 e_{,}^2)}{a_{,}^5} a a_{,} b_{5,1}$$

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[*] The numbers at the side serve to distinguish the arguments.

$$-\frac{1\,.\,3}{2\,.\,4\,a_{,}^{5}}\Big\{2a^4e^2+5a^2a_{,}^{2}(e^2+e_{,}^{2})+2a_{,}^{4}e_{,}^{2}\Big\}\,b_{5,2}$$

$$+\frac{1\,.\,3\,.\,2}{2\,.\,4\,a_{,}^{5}}(a^2e^2+a_{,}^{2}e_{,}^{2})\,b_{5,3}\Big\}\cos(2nt-2n_{,}t+2\varepsilon-2\varepsilon_{,})\qquad[2]$$

$$+m_{,}\Big\{-\frac{b_{1,3}}{a_{,}}+\frac{a\,a_{,}}{2\,a_{,}^{3}}\Big\{\sin^2\frac{\iota_{,}}{2}+\frac{e^2+e_{,}^{2}}{2}\Big\}\,b_{3,2}+\frac{3(a^2e^2+a_{,}^{2}e_{,}^{2})}{2\,.\,2\,a_{,}^{3}}\,b_{3,3}$$

$$+\frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2a_{,}^{2}(e^2+e_{,}^{2})}{a_{,}^{5}}\,b_{5,1}+\frac{1\,.\,3\,.\,2}{2\,.\,4}\,\frac{(a^2e^2+a_{,}^{2}e_{,}^{2})}{4a_{,}^{5}}\,aa_{,}b_{5,2}$$

$$-\frac{1\,.\,3}{2\,.\,4}\Big\{2a^4e^2+5a^2a_{,}^{2}(e^2+e_{,}^{2})+2a_{,}^{4}e_{,}^{2})\Big\}\,b_{5,3}\Big\}\cos(3nt-3n_{,}t+3\varepsilon-3\varepsilon_{,})\qquad[3]$$

$$+m_{,}\Big\{-\frac{b_{1,4}}{a_{,}}+\frac{a\,a_{,}}{2\,a_{,}^{3}}\Big\{\sin^2\frac{\iota_{,}}{2}+\frac{e^2+e_{,}^{2}}{2}\Big\}\,b_{3,3}+\frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2a_{,}^{2}(e^2+e_{,}^{2})}{a_{,}^{5}}\,b_{5,2}$$

$$+\frac{1\,.\,3\,.\,2}{2\,.\,4}\,\frac{(a^2e^2+a_{,}^{2}e_{,}^{2})}{a_{,}^{5}}\,aa_{,}b_{5,3}\Big\}\cos(4nt-4n_{,}t+4\varepsilon-4\varepsilon_{,})\qquad[4]$$

$$+m_{,}\Big\{-\frac{b_{1,5}}{a_{,}}+\frac{1\,.\,3\,.\,3}{2\,.\,4\,.\,2}\,\frac{a^2a_{,}^{2}(e^2+e_{,}^{2})}{a_{,}^{5}}\,b_{5,3}\Big\}\cos(5nt-5n_{,}t+5\varepsilon-5\varepsilon_{,})\qquad[5]$$

$$+m_{,}\Big\{-\frac{3\,a}{2\,a_{,}^{2}}+\frac{3\,a\,a'}{2\,a_{,}^{3}}\,b_{3,0}-\frac{a^2}{2\,a_{,}^{3}}\,b_{3,1}-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}\,b_{3,2}\Big\}\,e\cos(n_{,}t+\varepsilon_{,}-\varpi)\qquad[6]$$

$$+m_{,}\Big\{-\frac{a^2}{a_{,}^{3}}\,b_{3,0}+\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,1}\Big\}\,e\cos(nt+\varepsilon-\varpi)\qquad[7]$$

$$+m_{,}\Big\{\frac{1}{2}\,\frac{a}{a_{,}^{2}}-\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,0}-\frac{a^2}{2\,a_{,}^{3}}\,b_{3,1}+\frac{3}{2\,.\,2}\,\frac{a\,a_{,}}{a_{,}^{3}}\,b_{3,2}\Big\}\,e\cos(2nt-n_{,}t+2\varepsilon-\varepsilon_{,}-\varpi)\qquad[8]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}\,b_{3,1}-\frac{a^2}{2\,a_{,}^{3}}\,b_{3.2}+\frac{3}{2\,.\,2}\,\frac{a\,a_{,}}{a_{,}^{3}}\,b_{3,3}\Big\}\,e\cos(3nt-2n_{,}t+3\varepsilon-2\varepsilon_{,}-\varpi)\qquad[9]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}\,b_{3,2}-\frac{a^2}{2\,a_{,}^{3}}\,b_{3,3}\Big\}\,e\cos(4nt-3n_{,}t+4\varepsilon-3\varepsilon_{,}-\varpi)\qquad[10]$$

$$-m_{,}\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}\,b_{3,3}\,e\cos(5nt-4n_{,}t+5\varepsilon-4\varepsilon_{,}-\varpi)\qquad[11]$$

$$+m_{,}\Big\{\frac{3}{2\,.\,2}\,\frac{a\,a_{,}}{a_{,}^{3}}\,b_{3,1}-\frac{a^2}{2\,a_{,}^{3}}\,b_{3,2}-\frac{a\,a_{,}}{2\,.\,2\,a_{,}^{3}}\,b_{3,3}\Big\}\,e\cos(nt-2n_{,}t+\varepsilon-2\varepsilon_{,}+\varpi)\qquad[12]$$

$$+m_{,}\Big\{\frac{3}{2\,.\,2}\,\frac{a\,a_{,}}{a_{,}^{3}}\,b_{3,2}-\frac{a^2}{2\,a_{,}^{3}}\,b_{3,3}\Big\}\,e\cos(2nt-3n_{,}t+2\varepsilon-3\varepsilon_{,}+\varpi)\qquad[13]$$

$$+m_{,}\frac{3\,a\,a_{,}}{2\,.\,2\,a_{,}^{3}}\,b_{3,3}\,e\cos(3nt-4n_{,}t+3\varepsilon-4\varepsilon_{,}+\varpi)\qquad[14]$$

$$+m_{\prime}\left\{-\frac{a_{\prime}^2}{a_{\prime}^3}b_{3,0}-\frac{a\,a_{\prime}}{2\,a_{\prime}^3}b_{3,1}\right\}e_{\prime}\cos(n_{\prime}t+\varepsilon_{\prime}-\varpi_{\prime}) \qquad [15]$$

$$+m_{\prime}\left\{\frac{3\,a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,1}-\frac{a_{\prime}^2}{2\,a_{\prime}^3}b_{3,2}-\frac{a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,3}\right\}e_{\prime}\cos(2nt-n_{\prime}t+2\varepsilon-\varepsilon_{\prime}-\varpi_{\prime}) \qquad [17]$$

$$+m_{\prime}\left\{\frac{3\,a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,2}-\frac{a_{\prime}^2}{2\,a_{\prime}^3}b_{3,3}\right\}e_{\prime}\cos(3nt-2n_{\prime}t+3\varepsilon-2\varepsilon_{\prime}-\varpi_{\prime}) \qquad [18]$$

$$+m_{\prime}\frac{3\,a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,3}\,e_{\prime}\cos(4nt-3n_{\prime}t+4\varepsilon-3\varepsilon_{\prime}-\varpi_{\prime}) \qquad [19]$$

$$+m_{\prime}\left\{\frac{2\,a}{a_{\prime}^2}-\frac{a\,a_{\prime}}{2\,a_{\prime}^3}b_{3,0}-\frac{a_{\prime}^2}{2\,a_{\prime}^3}b_{3,1}+\frac{3}{2\,.\,2}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,2}\right\}e_{\prime}\cos(nt-2n_{\prime}t+\varepsilon-2\varepsilon_{\prime}+\varpi_{\prime}) \qquad [20]$$

$$+m_{\prime}\left\{-\frac{a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,1}-\frac{a_{\prime}^2}{2\,a_{\prime}^3}b_{3,2}+\frac{3}{2\,.\,2}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,3}\right\}e_{\prime}\cos(2nt-3n_{\prime}t+2\varepsilon-3\varepsilon_{\prime}+\varpi_{\prime}) \qquad [21]$$

$$+m_{\prime}\left\{-\frac{a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,2}-\frac{a_{\prime}^2}{2\,a_{\prime}^3}b_{3,3}\right\}e_{\prime}\cos(3nt-4n_{\prime}t+3\varepsilon-4\varepsilon_{\prime}+\varpi_{\prime}) \qquad [22]$$

$$-m_{\prime}\frac{a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,3}\,e_{\prime}\cos(4nt-5n_{\prime}t+4\varepsilon-5\varepsilon_{\prime}+\varpi_{\prime}) \qquad [23]$$

$$+m_{\prime}\left\{-\frac{a\,a_{\prime}}{2\,.\,8\,a_{\prime}^3}b_{3,1}-\frac{a^2}{2\,.\,4\,a_{\prime}^3}b_{3,2}-\frac{3}{2\,.\,8}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,3}-\frac{1\,.\,3\,.\,9}{2\,.\,4\,.\,2}\frac{a^2\,a_{\prime}^2}{a_{\prime}^5}b_{5,0}+\frac{1\,.\,3\,.\,3}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,1}\right.$$

$$\left.-\frac{1\,.\,3}{2\,.\,4\,.\,2}\frac{(2\,a^2-3\,a_{\prime}^3)}{a_{\prime}^5}a^2\,b_{5,2}-\frac{1\,.\,3}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,3}\right\}e^2\cos(2n_{\prime}t+2\varepsilon_{\prime}-2\varpi) \qquad [24]$$

$$+m_{\prime}\left\{\frac{a}{8\,a_{\prime}^2}-\frac{aa_{\prime}}{2\,.\,4\,a_{\prime}^3}b_{3,0}-\frac{a^2}{2\,.\,4\,a_{\prime}^3}b_{3,1}-\frac{3\,a\,a_{\prime}}{2\,.\,8\,a_{\prime}^3}b_{3,2}+\frac{1\,.\,3\,.\,6}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,0}\right.$$

$$-\frac{1\,.\,3\,a^2}{2\,.\,4\,.\,4}\frac{(4\,a^2+3\,a_{\prime}^2)}{a_{\prime}^5}b_{5,1}-\frac{1\,.\,3}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,2}$$

$$\left.-\frac{1\,.\,3}{2\,.\,4\,.\,4}\frac{a^2\,a_{\prime}^2}{a_{\prime}^5}b_{5,3}\right\}e^2\cos(nt+n_{\prime}t+\varepsilon+\varepsilon_{\prime}-2\varpi) \qquad [25]$$

$$+m_{\prime}\left\{-\frac{a^2}{2\,.\,2\,a_{\prime}^3}b_{3,0}-\frac{a\,a_{\prime}}{2\,.\,2\,a_{\prime}^3}b_{3,1}-\frac{1\,.\,3}{2\,.\,4}\frac{a^2\,(2\,a^2-3\,a_{\prime}^2)}{a_{\prime}^5}b_{5,0}\right.$$

$$\left.+\frac{1\,.\,3\,.\,2}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,1}-\frac{1\,.\,3\,.\,5}{2\,.\,4\,.\,2}\frac{a^2\,a_{\prime}^2}{a_{\prime}^5}b_{5,2}\right\}e^2\cos(2nt+2\varepsilon-2\varpi) \qquad [26]$$

$$+m_{\prime}\left\{\frac{3}{8}\frac{a}{a_{\prime}^2}-\frac{3}{8}\frac{a\,a_{\prime}}{a_{\prime}^3}b_{3,0}-\frac{a\,a_{\prime}}{2\,.\,4\,a_{\prime}^3}b_{3,1}-\frac{a^2}{2\,.\,8\,a_{\prime}^3}b_{3,2}-\frac{1\,.\,3\,.\,2}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,0}\right.$$

$$-\frac{1\,.\,3\,a^2}{2\,.\,4\,.\,4}\frac{(4\,a^2-5\,a_{\prime}^2)}{a_{\prime}^5}b_{5,1}+\frac{1\,.\,3\,.\,3}{2\,.\,4}\frac{a^3\,a_{\prime}}{a_{\prime}^5}b_{5,2}$$

$$-\frac{1.3.9}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\Big\}e^2\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-2\varpi) \qquad [27]$$

$$+m_{,}\Big\{-\frac{3\,a\,a_{,}}{2.8\,a_{,}^3}b_{3,1}-\frac{a^2}{2.4\,a_{,}^3}b_{3,2}-\frac{a\,a_{,}}{2.8\,a_{,}^3}b_{3,3}-\frac{1.3\,a^2a_{,}^2}{2.4.2\,a_{,}^5}b_{5,0}$$

$$-\frac{1.3}{2.4}\frac{a^3a_{,}}{a_{,}^5}b_{5,1}-\frac{1.3\,a^2}{2.4.2}\frac{(2\,a^2-3\,a_{,}^2)}{a_{,}^5}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{a^3a_{,}}{a_{,}^5}b_{5,3}\Big\}e^2\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-2\varpi) \qquad [28]$$

$$+m_{,}\Big\{-\frac{3\,a\,a_{,}}{2.8\,a_{,}^3}b_{3,2}-\frac{a^2}{2.4\,a_{,}^3}b_{3,3}-\frac{1.3}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}-\frac{1.3}{2.4}\frac{a^3a_{,}}{a_{,}^5}b_{5,2}$$

$$-\frac{1.3\,a^2}{2.4.2}\frac{(2\,a^2-3\,a_{,}^2)}{a_{,}^5}b_{5,3}\Big\}e^2\cos(5nt-3n_{,}t+5\varepsilon-3\varepsilon_{,}-2\varpi) \qquad [29]$$

$$+m_{,}\Big\{-\frac{3\,aa_{,}}{2.8\,a_{,}^3}b_{3,3}-\frac{1.3}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$-\frac{1.3}{2.4}\frac{a^3a_{,}}{a_{,}^5}b_{5,3}\Big\}e^2\cos(6nt-4n_{,}t+6\varepsilon-4\varepsilon_{,}-2\varpi) \qquad [30]$$

$$-m_{,}\frac{1.3}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\,e^2\cos(7nt-5n_{,}t+7\varepsilon-5\varepsilon_{,}-2\varpi) \qquad [31]$$

$$+m_{,}\Big\{-\frac{aa_{,}}{2.8\,a_{,}^3}b_{3,2}-\frac{a^2}{2.4\,a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}+\frac{1.3.3}{2.4}\frac{a^3a_{,}}{a_{,}^5}b_{5,2}$$

$$-\frac{1.3}{2.4}\frac{a^2}{2}\frac{(2\,a^2-3\,a_{,}^2)}{a_{,}^5}b_{5,3}\Big\}e^2\cos(nt-3n_{,}t+\varepsilon-3\varepsilon_{,}+2\varpi) \qquad [32]$$

$$-m_{,}\Big\{-\frac{aa_{,}}{2.8\,a_{,}^3}b_{3,3}-\frac{1.3.9}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{a^3a_{,}}{a_{,}^5}b_{5,3}\Big\}e^2\cos(2nt-4n_{,}t+2\varepsilon-4\varepsilon_{,}+2\varpi) \qquad [33]$$

$$-m_{,}\frac{1.3.9}{2.4.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\,e^2\cos(3nt-5n_{,}t+3\varepsilon-5\varepsilon_{,}+2\varpi) \qquad [34]$$

$$+m_{,}\Big\{-\frac{5}{2.2}\frac{a\,a_{,}}{a_{,}^3}b_{3,1}+\frac{1.3.2}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}+\frac{1.3.2}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}\,b_{5,1}$$

$$-\frac{1.3.5}{2.4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\Big\}ee_{,}\cos(nt-n_{,}t+\varepsilon-\varepsilon_{,}-\varpi+\varpi_{,}) \qquad [35]$$

$$+m_{,}\Big\{\frac{a}{a_{,}^2}-\frac{a\,a_{,}}{2.2\,a_{,}^3}b_{3,0}-\frac{9}{2.4}\frac{a\,a_{,}}{a_{,}^3}b_{3,2}-\frac{1.3.2}{2.4}\frac{(a^2+a_{,}^2)}{a_{,}^5}aa_{,}\,b_{5,0}$$

$$+\frac{1\cdot3}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}+\frac{1\cdot3\cdot3}{2\cdot4}\frac{a^2+a_{\prime}^2}{a_{\prime}^5}aa_{\prime}b_{5,2}$$

$$-\frac{1\cdot3\cdot9}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\Big\}ee_{\prime}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \qquad [36]$$

$$+m_{\prime}\Big\{-\frac{aa_{\prime}}{2\cdot4a_{\prime}^3}b_{3,1}-\frac{9}{2\cdot4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,3}-\frac{1\cdot3}{2\cdot4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,0}-\frac{1\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,1}+\frac{1\cdot3}{2\cdot4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}$$

$$+\frac{1\cdot3\cdot3}{2\cdot4}\frac{a^2+a_{\prime}^2}{a_{\prime}^5}aa_{\prime}b_{5,3}\Big\}ee_{\prime}\cos(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \qquad [37]$$

$$+m_{\prime}\Big\{-\frac{aa_{\prime}}{2\cdot4a_{\prime}^3}b_{3,2}-\frac{1\cdot3}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}-\frac{1\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,2}$$

$$+\frac{1\cdot3}{2\cdot4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\Big\}ee_{\prime}\cos(4nt-4n_{\prime}t+4\varepsilon-4\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \qquad [38]$$

$$+m_{\prime}\Big\{-\frac{aa_{\prime}}{2\cdot4a_{\prime}^3}b_{3,3}-\frac{1\cdot3}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}$$

$$-\frac{1\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,3}\Big\}ee_{\prime}\cos(5nt-5n_{\prime}t+5\varepsilon-5\varepsilon_{\prime}-\varpi+\varpi_{\prime}) \qquad [39]$$

$$-m_{\prime}\frac{1\cdot3}{2\cdot4}\frac{a^2a_{\prime}^2}{2a_{\prime}^5}b_{5,3}\cos(6nt-6n_{\prime}t-\varpi+\varpi_{\prime}) \qquad [40]$$

$$+m_{\prime}\Big\{-\frac{9}{2\cdot2}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,0}-\frac{aa_{\prime}}{2\cdot4a_{\prime}^3}b_{3,2}+\frac{1\cdot3\cdot6}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,0}-\frac{1\cdot3\cdot7}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}$$

$$-\frac{1\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}a_{\prime}a\,b_{5,2}-\frac{1\cdot3}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\Big\}ee_{\prime}\cos(\varpi-\varpi_{\prime}) \qquad [41]$$

$$+m_{\prime}\Big\{-\frac{9}{2\cdot4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,1}-\frac{aa_{\prime}}{2\cdot4a_{\prime}^3}b_{3,3}-\frac{1\cdot3\cdot9}{2\cdot4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,0}+\frac{1\cdot3\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,1}$$

$$+\frac{1\cdot3}{2\cdot4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}-\frac{1\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,3}\Big\}ee_{\prime}\cos(nt-n_{\prime}t+\varepsilon-\varepsilon_{\prime}+\varpi-\varpi_{\prime}) \qquad [42]$$

$$+m_{\prime}\Big\{-\frac{9}{2\cdot4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,2}-\frac{1\cdot3\cdot9}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,1}+\frac{1\cdot3\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,2}$$

$$+\frac{1\cdot3}{2\cdot4}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,3}\Big\}ee_{\prime}\cos(2nt-2n_{\prime}t+2\varepsilon-2\varepsilon_{\prime}+\varpi-\varpi_{\prime}) \qquad [43]$$

$$+m_{\prime}\Big\{-\frac{9}{2\cdot4}\frac{aa_{\prime}}{a_{\prime}^3}b_{3,3}-\frac{1\cdot3\cdot9}{2\cdot4\cdot2}\frac{a^2a_{\prime}^2}{a_{\prime}^5}b_{5,2}$$

$$+\frac{1\cdot3\cdot3}{2\cdot4}\frac{(a^2+a_{\prime}^2)}{a_{\prime}^5}aa_{\prime}b_{5,3}\Big\}ee_{\prime}\cos(3nt-3n_{\prime}t+3\varepsilon-3\varepsilon_{\prime}+\varpi-\varpi_{\prime}) \qquad [44]$$

$$-m_{,}\frac{1.3.9}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\,ee_{,}\cos(4nt-4n_{,}t+4\varepsilon-4\varepsilon_{,}+\varpi-\varpi_{,}) \qquad [45]$$

$$\begin{aligned}&+m_{,}\left\{-\frac{3a}{a_{,}^2}+\frac{3aa_{,}}{2.2a_{,}^3}b_{3,0}+\frac{3aa_{,}}{2.4a_{,}^3}b_{3,2}+\frac{1.3.2}{1.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}b_{5,0}-\frac{1.3.11}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,1}\right.\\&\left.+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}b_{5,2}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\right\}ee_{,}\cos(2n_{,}t+2\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [46]\end{aligned}$$

$$\begin{aligned}&+m_{,}\left\{\frac{3}{2.2}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,1}-\frac{1.3.14}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,0}+\frac{1.3.2}{2.4}\,\frac{(a^2+a_{,}^2)}{a_{,}^5}\,aa_{,}b_{5,1}\right.\\&\qquad\left.+\frac{1.3.3}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,2}\right\}ee_{,}\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [47]\end{aligned}$$

$$\begin{aligned}&+m_{,}\left\{\frac{3aa_{,}}{2.2a_{,}^3}b_{3,0}+\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,2}+\frac{1.3.2}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}b_{5,0}-\frac{1.3.11}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,1}\right.\\&\qquad\left.+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}b_{5,2}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\right\}ee_{,}\cos(2nt+2\varepsilon-\varpi-\varpi_{,}) \qquad [48]\end{aligned}$$

$$\begin{aligned}&+m_{,}\left\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,1}+\frac{3.aa_{,}}{2.4a_{,}^3}\,b_{3,3}+\frac{1.3.3}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,0}\right.\\&\qquad+\frac{1.3}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}b_{5,1}-\frac{1.3.7}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,2}\\&\qquad\left.+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}b_{5,3}\right\}ee_{,}\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [49]\end{aligned}$$

$$\begin{aligned}&+m_{,}\left\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,2}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,1}+\frac{1.3}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}b_{5,2}\right.\\&\qquad\left.-\frac{1.3.7}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\right\}ee_{,}\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [50]\end{aligned}$$

$$\begin{aligned}&+m_{,}\left\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,3}+\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,aa_{,}b_{5,2}\right.\\&\qquad\left.+\frac{1.3}{2.4}\,\frac{(3a^2-a_{,}^2)}{a_{,}^5}\,aa_{,}b_{5,3}\right\}ee_{,}\cos(5nt-3n_{,}t+5\varepsilon-3\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [51]\end{aligned}$$

$$+m_{,}\frac{1.3.3}{2.4.2}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,3}\,ee_{,}\cos(6nt-4n_{,}t+6\varepsilon-4\varepsilon_{,}-\varpi-\varpi_{,}) \qquad [52]$$

$$\begin{aligned}&+m_{,}\left\{\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,1}+\frac{3}{2.4}\,\frac{aa_{,}}{a_{,}^3}\,b_{3,3}+\frac{1.3.3}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,0}\right.\\&\qquad+\frac{1.3}{2.4}\,\frac{(3a_{,}^2-a^2)}{a_{,}^5}\,aa_{,}b_{5,1}-\frac{1.3.7}{2.4}\,\frac{a^2a_{,}^2}{a_{,}^5}\,b_{5,2}\end{aligned}$$

$$+\frac{1\cdot 3}{2\cdot 4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,3}\Big\}ee_{,}\cos(nt-3n_{,}t+\varepsilon-3\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [53]$$

$$+m_{,}\Big\{\frac{3}{2\cdot 4}\frac{a\,a_{,}}{a_{,}^3}b_{3,1}+\frac{3}{2\cdot 4}\frac{a\,a_{,}}{a_{,}^3}b_{3,2}+\frac{1\cdot 3\cdot 3}{2\cdot 4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}$$

$$+\frac{1\cdot 3}{2\cdot 4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,1}+\frac{1\cdot 3}{2\cdot 4}\frac{(3a^2-a_{,}^2)}{a_{,}^5}aa_{,}b_{5,2}$$

$$-\frac{1\cdot 3\cdot 7}{2\cdot 4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\Big\}ee_{,}\cos(2nt-4n_{,}t+2\varepsilon-4\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [54]$$

$$+m_{,}\Big\{\frac{3}{2\cdot 4}\frac{a\,a_{,}}{a_{,}^3}b_{3,3}+\frac{1\cdot 3\cdot 3}{2\cdot 4\cdot 2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$+\frac{1\cdot 3}{2\cdot 4}\frac{(3a_{,}^2-a^2)}{a_{,}^5}aa_{,}b_{5,3}\Big\}ee_{,}\cos(3nt-5n_{,}t+3\varepsilon-5\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [55]$$

$$+m_{,}\frac{1\cdot 3\cdot 3}{2\cdot 4\cdot 2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}ee_{,}\cos(4nt-6n_{,}t+4\varepsilon-6\varepsilon_{,}+\varpi+\varpi_{,}) \qquad [56]$$

$$+m_{,}\Big\{-\frac{a_{,}^2}{2\cdot 2a_{,}^3}b_{3,0}-\frac{a\,a_{,}}{2\cdot 2a_{,}^3}b_{3,1}-\frac{1\cdot 3}{2\cdot 4}\frac{(2a_{,}^2-3a^2)}{a_{,}^5}a_{,}^2b_{5,0}$$

$$+\frac{1\cdot 3\cdot 2}{2\cdot 4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,1}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}\Big\}e_{,}^2\cos(2n_{,}t+2\varepsilon_{,}-2\varpi_{,}) \qquad [57]$$

$$+m_{,}\Big\{\frac{a}{8a_{,}^2}-\frac{a\,a_{,}}{2\cdot 4a_{,}^3}b_{3,0}-\frac{a_{,}^2}{2\cdot 4a_{,}^3}b_{3,1}-\frac{3a\,a_{,}}{2\cdot 8a_{,}^3}b_{3,2}+\frac{1\cdot 3\cdot 6}{2\cdot 4}\frac{a_{,}^3a}{a_{,}^5}b_{5,0}$$

$$-\frac{1\cdot 3}{2\cdot 4\cdot 4}\frac{(4a^2+3a_{,}^2)}{a_{,}^5}a_{,}^2b_{5,1}-\frac{1\cdot 3}{2\cdot 4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,2}$$

$$-\frac{1\cdot 3}{2\cdot 4\cdot 4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,3}\Big\}e_{,}^2\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-2\varpi_{,}) \qquad [58]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2\cdot 8a_{,}^3}b_{3,1}-\frac{a_{,}^2}{2\cdot 4a_{,}^3}b_{3,2}-\frac{3}{2\cdot 8}\frac{aa_{,}}{a_{,}^3}b_{3,3}-\frac{1\cdot 3\cdot 9}{2\cdot 4\cdot 2}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,0}+\frac{1\cdot 3\cdot 3}{2\cdot 4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,1}$$

$$-\frac{1\cdot 3}{2\cdot 4\cdot 2}\frac{(2a_{,}^2-3a^2)}{a_{,}^5}a_{,}^2b_{5,2}-\frac{1\cdot 3}{2\cdot 4}\frac{a\,a_{,}^3}{a_{,}^5}b_{5,3}\Big\}e_{,}^2\cos(2nt+2\varepsilon-2\varpi_{,}) \qquad [59]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2\cdot 8a_{,}^3}b_{3,2}-\frac{a_{,}^2}{2\cdot 4a_{,}^3}b_{3,3}-\frac{1\cdot 3\cdot 9}{2\cdot 4\cdot 4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,1}+\frac{1\cdot 3\cdot 3}{2\cdot 4}\frac{a_{,}^3a}{a_{,}^5}b_{5,2}$$

$$-\frac{1\cdot 3}{2\cdot 4\cdot 2}\frac{(2a_{,}^2-3a^2)}{a_{,}^5}a_{,}^2b_{5,3}\Big\}e_{,}^2\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-2\varpi_{,}) \qquad [60]$$

$$+m_{,}\Big\{-\frac{a\,a_{,}}{2\cdot 8a_{,}^3}b_{3,3}-\frac{1\cdot 3\cdot 9}{2\cdot 4\cdot 4}\frac{a^2a_{,}^2}{a_{,}^5}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{a_{,}^{3}a}{a_{,}^{5}}b_{5,3}\Big\}\, e_{,}^{2}\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-2\varpi_{,}) \qquad [61]$$

$$-m_{,}\frac{1.3.9}{2.4.4}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,3}e_{,}^{2}\cos(5nt-3n_{,}t+5\varepsilon-3\varepsilon_{,}-2\varpi_{,}) \qquad [62]$$

$$+m_{,}\Big\{\frac{27}{8}\frac{a}{a_{,}^{2}}-\frac{3\,a\,a_{,}}{2.4\,a_{,}^{3}}b_{3,0}-\frac{a_{,}^{2}}{2.4\,a_{,}^{3}}b_{3,1}-\frac{aa_{,}}{2.8\,a_{,}^{3}}b_{3,2}-\frac{1.3.2}{2.4}\frac{a_{,}^{3}a}{a_{,}^{5}}b_{5,0}$$

$$-\frac{1.3}{2.4.4}\frac{(4\,a_{,}^{2}-5\,a^{2})}{a_{,}^{5}}a_{,}^{2}b_{5,1}+\frac{1.3.3}{2.4}\frac{aa_{,}^{3}}{a_{,}^{5}}b_{5,2}$$

$$-\frac{1.3.9}{2.4.4}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,3}\Big\}\, e_{,}^{2}\cos(nt-3n_{,}t+\varepsilon-3\varepsilon_{,}+2\varpi_{,}) \qquad [63]$$

$$+m_{,}\Big\{-\frac{3\,aa_{,}}{2.8\,a_{,}^{3}}b_{3,1}-\frac{a_{,}^{2}}{2.4\,a_{,}^{3}}b_{3,2}-\frac{aa_{,}}{2.8\,a_{,}^{3}}b_{3,3}-\frac{1.3}{2.4.2}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,0}$$

$$-\frac{1.3}{2.4}\frac{aa_{,}^{3}}{a_{,}^{5}}b_{5,1}-\frac{1.3}{2.4.2}\frac{(2\,a_{,}^{2}-3\,a^{2})}{a_{,}^{5}}a_{,}^{2}b_{5,2}$$

$$+\frac{1.3.3}{2.4}\frac{aa_{,}^{3}}{a_{,}^{5}}b_{5,3}\Big\}\, e_{,}^{2}\cos(2nt-4n_{,}t+2\varepsilon-4\varepsilon_{,}+2\varpi_{,}) \qquad [64]$$

$$+m_{,}\Big\{-\frac{3\,aa_{,}}{2.8\,a_{,}^{3}}b_{3,2}-\frac{a_{,}^{2}}{2.4\,a_{,}^{3}}b_{3,3}-\frac{1.3}{2.4.4}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,1}-\frac{1.3}{2.4}\frac{aa_{,}^{3}}{a_{,}^{5}}b_{5,2}$$

$$-\frac{1.3}{2.4.2}\frac{(2\,a_{,}^{2}-3\,a^{2})}{a_{,}^{5}}a_{,}^{2}b_{5,3}\Big\}\, e_{,}^{2}\cos(3nt-5n_{,}t+3\varepsilon-3\varepsilon_{,}+2\varpi_{,}) \qquad [65]$$

$$+m_{,}\Big\{-\frac{3}{2.8}\frac{aa_{,}}{a_{,}^{3}}b_{3,3}-\frac{1.3}{2.4.4}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,2}$$

$$-\frac{1.3}{2.4}\frac{aa_{,}^{3}}{a_{,}^{5}}b_{5,3}\Big\}\, e_{,}^{2}\cos(4nt-6n_{,}t+4\varepsilon-6\varepsilon_{,}+2\varpi_{,}) \qquad [66]$$

$$-m_{,}\frac{1.3}{2.4.4}\frac{a^{2}a_{,}^{2}}{a_{,}^{5}}b_{5,3}e_{,}^{2}\cos(5nt-7n_{,}t+5\varepsilon-7\varepsilon_{,}+2\varpi_{,}) \qquad [67]$$

$$+m_{,}\Big\{\frac{a}{a_{,}^{2}}\Big(1+\frac{3}{2}e_{,}^{2}\Big)-\frac{aa_{,}}{a_{,}^{3}}b_{3,0}\Big\}\sin^{2}\frac{\iota_{,}}{2}\cos(nt+n_{,}t+\varepsilon+\varepsilon_{,}-2\nu_{,}) \qquad [68]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,1}\sin^{2}\frac{\iota_{,}}{2}\cos(2n_{,}t+2\varepsilon_{,}-2\nu_{,}) \qquad [69]$$

$$m\,\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,1}\sin^{2}\frac{\iota_{,}}{2}\cos(2nt+2\varepsilon-2\nu_{,}) \qquad [70]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}b_{3,2}\sin^{2}\frac{\iota_{,}}{2}\cos(nt+3n_{,}t+\varepsilon-3\varepsilon_{,}+2\nu_{,}) \qquad [71]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,2}\sin^2\frac{\iota_{,}}{2}\cos(3nt-n_{,}t+3\varepsilon-\varepsilon_{,}-2\nu_{,}) \qquad [72]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,3}\sin^2\frac{\iota_{,}}{2}\cos(2nt-4n_{,}t+2\varepsilon-4\varepsilon_{,}+2\nu_{,}) \qquad [73]$$

$$-m_{,}\frac{a\,a_{,}}{2\,a_{,}^{3}}\,b_{3,3}\sin^2\frac{\iota_{,}}{2}\cos(4nt-2n_{,}t+4\varepsilon-2\varepsilon_{,}-2\nu_{,}) \qquad [74]$$

In the development of *R*, I have supposed $\iota = 0$, so that $\iota_{,}$ is the angle contained between the orbits of the planets P and $P_{,}$, or P_1 and P_2; in the general case, when ι_1, and ι_2, are retained, $\cos\iota_{,} = \cos\iota_1\cos\iota_2 + \sin\iota_1\sin\iota_2\cos(\iota_1-\iota_2)$, ι_1 and ι_2, being the inclinations of the orbits of the planets P_1, P_2 to any plane $x\,y$, of which the direction is arbitrary,

$$r^{\backprime}r_{,}^{\backprime}\sin(\lambda^{\backprime}-\lambda_{,}^{\backprime}) = r_1r_2\Big\{\cos^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1-\lambda_2)$$

$$-\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1+\lambda_2-2\,\nu_1)+\sin^2\frac{\iota_2}{2}\cos^2\frac{\iota_1}{2}\sin(\lambda_1+\lambda_2-2\,\nu_2)$$

$$-\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\sin(\lambda_1-\lambda_2-2\,\nu_1+2\,\nu_2)\Big\}$$

$$r^{\backprime}r_{,}^{\backprime}\Big\{\cos(\lambda^{\backprime}-\lambda_{,}^{\backprime})+s\,s_{,}\Big\} = r_1r_2\Big\{\cos^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\cos(\lambda_1-\lambda_2)$$

$$+\sin^2\frac{\iota_1}{2}\cos^2\frac{\iota_2}{2}\cos(\lambda_1+\lambda_2-2\,\nu_1)+\sin^2\frac{\iota_2}{2}\cos^2\frac{\iota_1}{2}\cos(\lambda_1+\lambda_2-2\,\nu_2)$$

$$+\sin^2\frac{\iota_1}{2}\sin^2\frac{\iota_2}{2}\cos(\lambda_1-\lambda_2-2\,\nu_1+2\,\nu_2)$$

$$+\sin\iota_1\sin\iota_2\cos(\lambda_1-\lambda_2-\nu_1+\nu_2)-\sin\iota_1\sin\iota_2\cos(\lambda_1+\lambda_2-\nu_1-\nu_2)\Big\}$$

ERRATA.

In page 330, *for* ι and ν, *read* $\iota_{,}$ and $\nu_{,}$.